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THE TRANSVERSE VELOCITY GRADIENT NEAR THE MOUTHS OF PIPES IN WHICH AN ALTERNATING OR CONTINUOUS FLOW OF AIR IS ESTABLISHED

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ABSTRACT. A comparison is made of the alternating and direct flow of air near the mouths of tubes of various sections, by traversal of a hot-wire anemometer across the tube. After a correction for the use of the hot wire in the neighbourhood of the walls of the tubes has been made, the theoretical and observed gradients of mean velocity across the sections are compared. In the alternating flow a peak of high average velocity is found near the walls of the pipe, but in one-way flow this annular peak is absent, the velocity falling continuously from the centre of the tube to within a short distance of the walls according to the law $v = ay^q$, where q varies from $1/2$ for stream-line flow up to $1/7$ for turbulent flow. The existence of a layer of laminar flow close to the walls, when the main body of air is in turbulent motion, is demonstrated.

§ 1. INTRODUCTION

IN measuring the velocity in the alternating flow of air in the orifice of a resonating Helmholtz resonator one of us has shown that, owing to inertia, the velocity in annuli remote from the centre of the orifice was much greater than at the centre itself, except very close to the walls of the orifice, where friction caused a rapid drop down to zero velocity. At the end of the paper* in which this effect was described appeared a few measurements of the velocity across the end of a pipe in which alternating flow of low frequency was produced by a piston oscillating in the pipe†. Warren‡ has since pointed out the analogy between the flow in such an orifice and the motion of a flexible diaphragm oscillating in the

* *Proc. Phys. Soc.* **40**, 206 (1928). The following errata in that paper should be noted: p. 208, Table I, for " $n = 1000$ " read " $n = 1100$ "; p. 212, a minus sign is to be attributed to v_2 , so that a minus sign is to be placed before the second term of the equation following, (7).

† Just recently, Carrière, in the *Journal de Physique*, **10**, 198 (1929), has confirmed these measurements by observation of the motion of microscopic particles in a sounding pipe.

‡ *Proc. Phys. Soc.* **40**, 296 (1928).

orifice. The experiments described in this paper were undertaken in order to examine the alternating flow of air in a pipe in more detail, and to compare it with continuous (one-way) flow.

§ 2. APPARATUS AND RESULTS FOR ALTERNATING FLOW

The apparatus is shown in Fig. 1. The alternating flow was produced by an electric motor oscillating a disc on the end of a fairly long crank with a stroke which could be varied at will. The frequency was regulated by observation of a strobo-

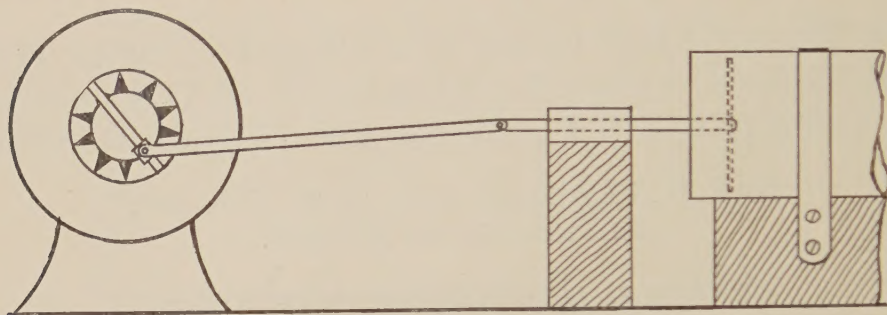


Fig. 1. Diagram of apparatus.

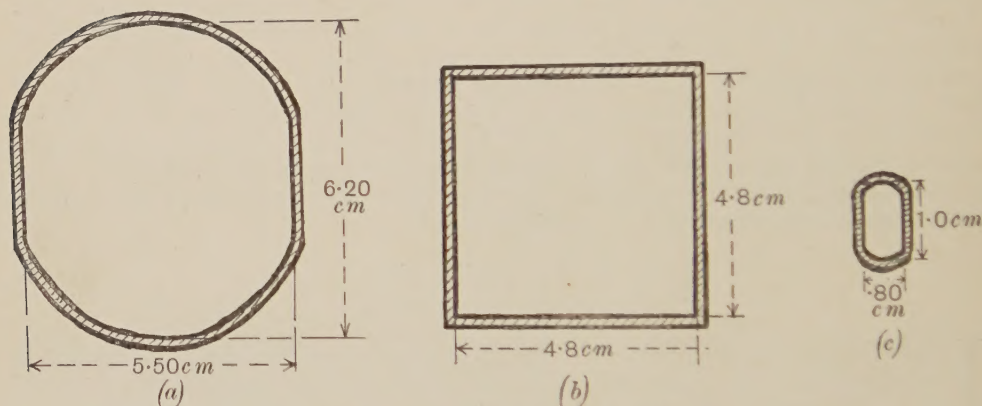


Fig. 2. Sections of tubes employed.

scopic disc on the motor which was illuminated by a neon tube at 50 ~. The piston moved to and fro near one end of a tube 33.0 cm. long, which it just cleared. The hot wire anemometer consisted of a piece of 0.002 in. nickel wire, 1.0 cm. long, supported vertically on a long fork attached to a travelling micrometer, so that it could be traversed across the tube at any section between the orifice and the piston. Two tubes were used, (a) one of circular section, 6.20 cm. in diameter, but with two "flats" formed at the sides to be parallel to the vertical hot wire; (b) a tube of square section 4.8 cm. wide, Fig. 2.

The wire was supplied with a current of 0.20 amp., and the change of resistance corresponding to a given average wind speed was measured on a Kelvin double

bridge. In this case the wire was calibrated by oscillating it on the piston itself at various amplitudes, and at the same frequencies as those used in the research. The calibration curve of resistance against maximum velocity at a given frequency

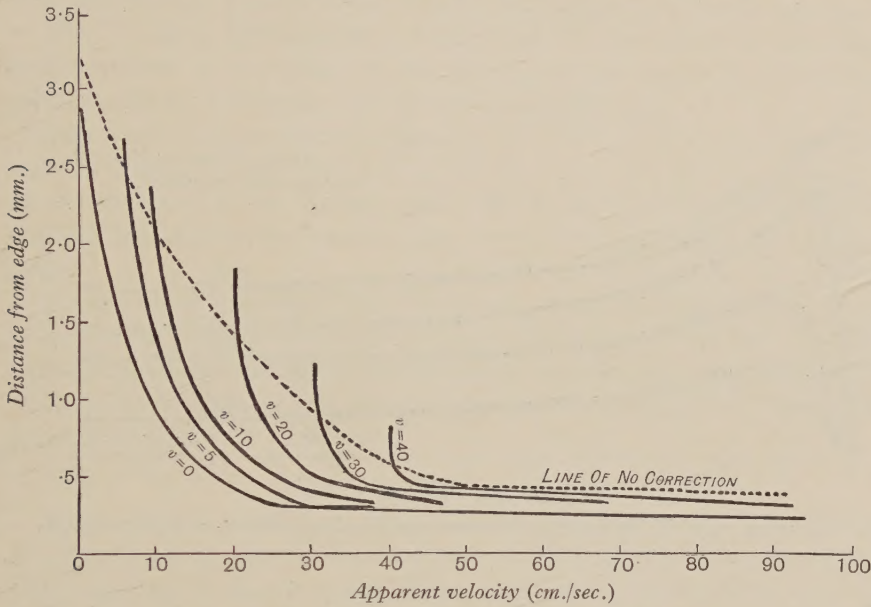


Fig. 3. Relation between true velocity v and apparent velocity.

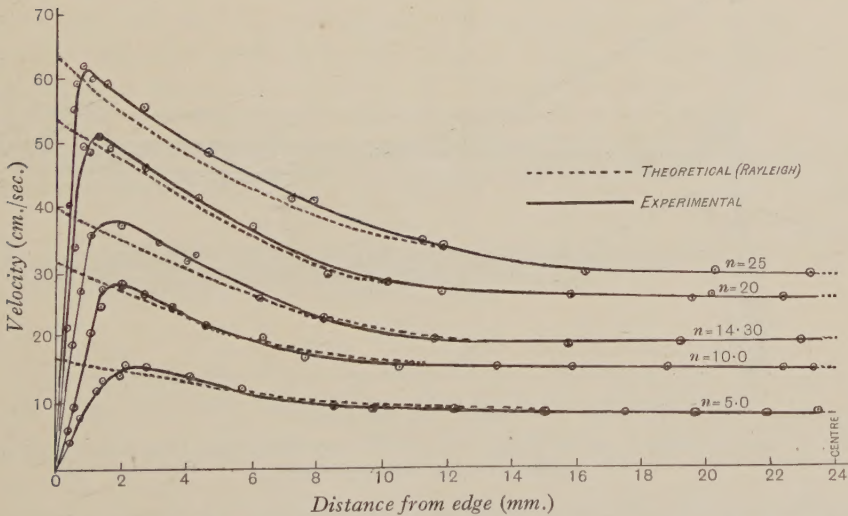


Fig. 4. Traverses 1 mm. inside opening of square section tubes. Length of stroke 0.95 cm.

was of the now familiar parabolic type. In the use of the hot wire close to a large metallic surface an error arises in the estimation of velocity due to the loss of heat by conduction across an intervening thin film of air. A correction for this at

steady velocities is available from some experiments published elsewhere*, in which a specimen of the wire was whirled on a fork round a brass cylinder of large diameter, at various small distances from the brass surface. From the resistance at

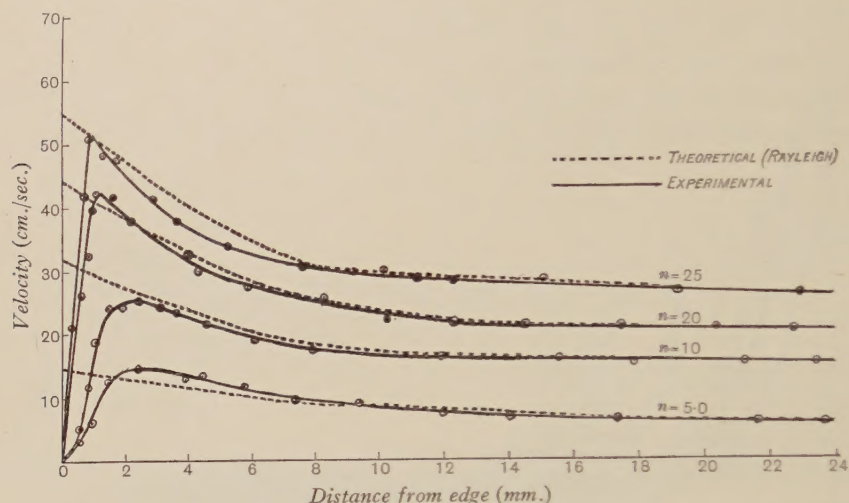


Fig. 5. Traverses 1 mm. inside opening of round section pipe. Length of stroke 0.95 cm.

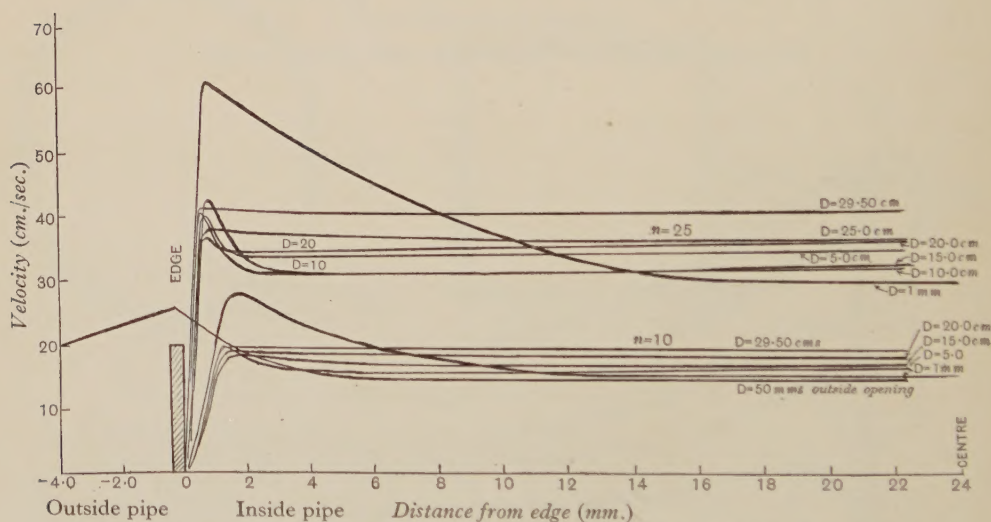


Fig. 6. Traverses across square section pipe at different distances D cm. from opening. Piston 31.0 cm. from opening.

a number of whirling speeds the apparent velocity from the ordinary calibration curve is read off. This has to be correlated with the true velocity, i.e. with the known velocity of the wire through the air round the cylinder. Fig. 3 shows such a correction curve of apparent to true velocity for a 0.002 in. wire taking 0.20 amp.

* Piercy and Richardson, *Aero. Res. Comm. R. and M.* No. 1229, 1929.

This was used on the velocities close to the walls of the tube in the present research, on the assumption that the average velocity in alternating flow would be affected similarly to a steady velocity of the same amount. A justification for this procedure is obtained in the passage of the velocity through zero at the boundary.

Figs. 4 and 5, corresponding to Fig. 9 for the round tube in the former paper, show traverses in the plane of the opening of the square and round tubes at a constant stroke of 0.95 cm., but at various frequencies n .

Fig. 6 shows traverses at frequencies of $25 \sim$ and $10 \sim$ respectively, at various sections down the tube from just outside the open end; it shows how the annular effect diminishes as the distance from the open end increases, the velocity being uniform (apart from the sudden drop in the boundary layer) just in front of the piston.

In one set of traverses a large flange was placed flush with the end of the tube, to reduce the possible solid angle of spreading by nearly one half. In confirmation of our suggestion that the inertia is thereby affected, the annular effect was much less pronounced, and at a distance 0.5 cm. outside the plane of opening the annular effect was completely eliminated.

§ 3. THEORY OF ALTERNATING FLOW

Rayleigh* has given the equation for the potential due to a flexible membrane with the condition that the velocity potential over the membrane shall be uniform. This equation is, however, insoluble unless the frequency term be neglected, in which case a solution corresponding to the theoretical steady flow through an orifice is obtained†. In order to retain the frequency in the solution the analogy of the membrane may be retained, but the membrane must be imagined to have a variable resistance over its surface, due to inertia. This resistance will be in phase with the velocity, but will decrease from the centre outwards, owing to the radial leakage of air from the confines of the orifice.

Sextl† has examined a number of cases of flow in tubes having axial symmetry, taking the general equations of flow of Stokes, with the stream function Ψ defined by

$$u = r^{-1} \delta \Psi / \delta l, \quad v = -r^{-1} \delta \Psi / \delta r$$

(where u is the radial, v the axial velocity, r the distance from the axis, l the length of the tube), in cylindrical co-ordinates, thus:

$$\frac{\delta}{\delta t} D \cdot \Psi + \frac{1}{r} \frac{\delta \Psi}{\delta l} \cdot \frac{\delta}{\delta r} D \Psi - \frac{1}{r} \frac{\delta \Psi}{\delta r} \cdot \frac{\delta}{\delta l} D \Psi - \frac{2}{r^2} \frac{\delta \Psi}{\delta l} \cdot D \Psi = \nu D D \Psi \quad \dots (1)$$

where ν is the kinematic viscosity and D is Stokes's operator, so that

$$D = \frac{\delta^2}{\delta r^2} - \frac{1}{r} \frac{\delta}{\delta r} + \frac{\delta^2}{\delta l^2}.$$

If we first consider the motion to be unchanged along the tube, i.e. independent of l , then (1) becomes

$$\left(D - \frac{1}{\nu} \frac{\delta}{\delta t} \right) \cdot D \Psi = 0,$$

* *Sound*, p. 176; quoted by Warren, *loc. cit.*

† Cf. Lamb, *Hydrodynamics*, § 102.

† *Ann. d. Physik*, 87, 570 (1928).

which, if Ψ is simple harmonic and $= e^{i\beta t}$, leads to two equations:

$$\frac{\delta^2}{\delta r^2} \Psi - \frac{1}{r} \frac{\delta}{\delta r} \Psi = 0$$

$$\frac{\delta^2}{\delta r^2} \Psi - \frac{1}{r} \frac{\delta}{\delta r} \Psi - \frac{i\beta}{v} \Psi = 0,$$

of which the solution (since v must be finite when $r = 0$, but be zero when $r = a$, where a is the radius of the pipe) is

$$v = -\frac{1}{r} \frac{\delta \Psi}{\delta r} = c \left\{ J_0 \left(\sqrt{\frac{-i\beta}{v}} a \right) - J_0 \left(\sqrt{\frac{-i\beta}{v}} r \right) \right\} e^{i\beta t}.$$

In terms of Kelvin's "ber" and "bei" functions, this is equivalent to

$$v = c (\text{ber } \alpha a + i \text{bei } \alpha a - \text{ber } \alpha r - i \text{bei } \alpha r) e^{i\beta t},$$

where α is put for $\beta^{\frac{1}{2}} v^{-\frac{1}{2}}$.

Finally, the maximum velocity in the simple harmonic motion at any radius is given by

$$\bar{v} = c_1 \{ (1 - \text{ber } \alpha r \cdot \text{ber}^{-1} \alpha a) \cdot \text{ber } \alpha a \cdot \sin \beta t + i (1 - \text{bei } \alpha r \cdot \text{bei}^{-1} \alpha a) \text{bei } \alpha a \cdot \cos \beta t \} = f(r) \dots\dots(2).$$

When α is small this gives nearly the Poiseuille distribution of velocity, but if α is given an increasing value corresponding to increasing frequency and to predomination of inertia over viscosity, substitution of the ber and bei functions from tables gives more and more pronounced peaks with fall to zero velocity at the walls, in correspondence with our curves.

The above treatment neglects the change in the distribution across a section which takes place as one recedes to the piston. Assuming an exponential change, some such equation as

$$\bar{v} = f(r) \{ (\kappa f(r)^{-1} - 1) e^{\gamma r} + 1 \} \dots\dots(3)^*$$

would give the necessary uniform velocity at the piston, and exhibit the change due to distance from the surface of the piston. If, to get the mean square velocity at any radius, we add the squares of the coefficients of $\sin \beta t$ and $\cos \beta t$ in (2) and substitute the known expansions of the ber and bei functions, i.e.

$$\text{ber } x = 1 - \frac{x^2}{2^2 \cdot 4^2} + \frac{x^8}{2^2 \cdot 4^2 \cdot 6^2 \cdot 8^2} - \dots,$$

$$\text{bei } x = \frac{x^2}{2^2} - \frac{x^4}{2^2 \cdot 4^2 \cdot 6^2} + \dots,$$

we shall obviously get for $\bar{v}$ a series of ascending even powers of r/a , when the latter is small. This is interesting, for Rayleigh, in an appendix to his *Sound*, tested an arbitrary distribution of velocity across the mouth of the orifice of a resonator of the form $\bar{v} = \bar{v}_0 (1 + \mu r^2/a^2 + \mu' r^4/a^4)$, where $\bar{v}_0$ is the central velocity, r is the distance of the point considered, and a is the radius of the edge of the flanged orifice, and he then determined μ and μ' so as to make the whole energy minimum. The velocity at the mouth being given by the above expression, that at infinity

* We are indebted to Mr A. M. Cassie, M.A., B.Sc., for suggesting this equation.

outside the tube being zero, and that at the piston ($x = -l$) being constant across the tube, then the resistance of the orifice is determined as the ratio of the difference of hydrodynamic potential across it to the current or velocity produced. For minimum energy ($\propto \Sigma v^2$) this resistance must be a maximum, a condition which

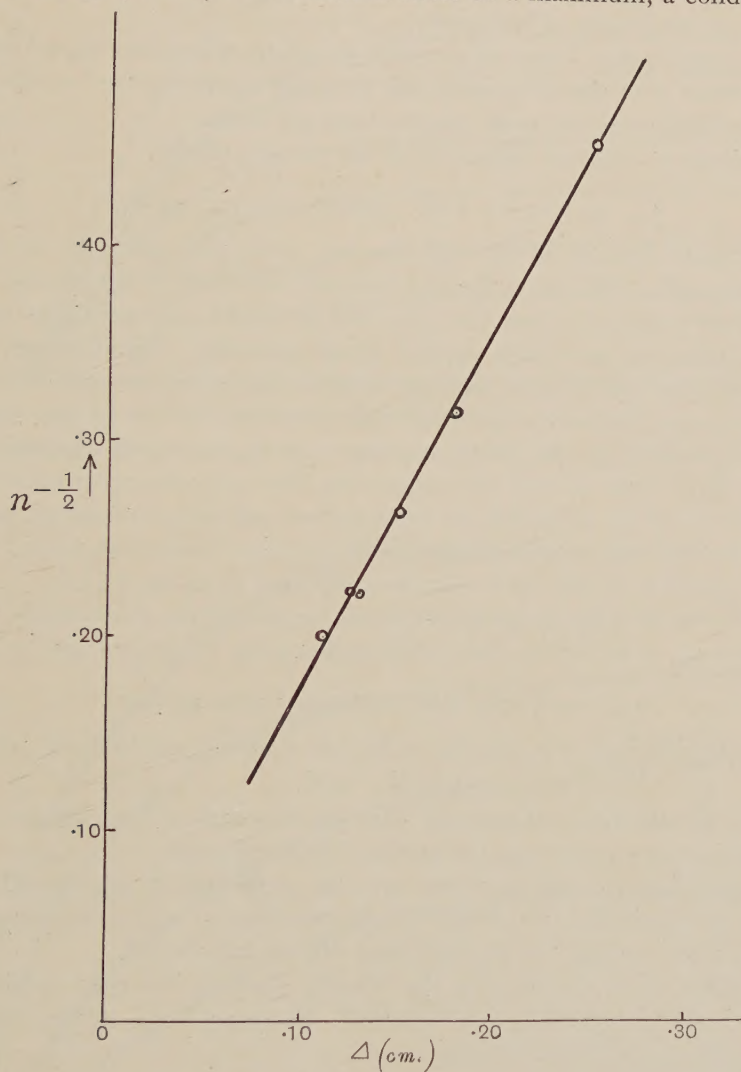


Fig. 7.

allows one to determine μ and μ' . This resistance Rayleigh co-ordinated with the "end correction," i.e. the distance from the end of the tube to the centre of the radial motion in the surroundings, and found that the best values of μ and μ' to fit the known practical end correction were 0 and 1.103 respectively.

Curiously enough, the expression

$$\bar{v} = v_0 (1 + 1.1 r^4/a^4)$$

fits our experimental curves for the unflanged tube at the orifice very well (cf. Figs. 4 and 5, dotted lines). For a flanged tube μ' is considerably less. Also μ' diminishes to zero as one approaches the piston.

At these low frequencies a change in frequency merely increases every ordinate in the ratio of the change of frequency.

In the former paper it was shown that we might reasonably expect $\Delta n^{\frac{1}{2}}$ to be constant, where n is the frequency and Δ is the width of the boundary layer, taken as the distance of the peak velocity from the edge.

Fig. 7 shows $n^{-\frac{1}{2}}$ plotted against Δ for the present results.

§ 4. RESULTS FOR CONTINUOUS FLOW

A continuous flow of air through the two tubes was produced by a blower operating through a diffuser at the end formerly occupied by the piston. The hot wire was then traversed across the open end as before, and velocities calculated with the conduction correction applied when necessary. Fig. 8 shows a set of curves so obtained for different average velocities in the square-section tube.

It was expected that at the lowest of these velocities the flow in such wide tubes would be turbulent for Reynolds' number = 1000, so to get comparison with stream-line flow a tube of smaller (oval) section, Fig. 2 (c), was constructed, in which the hot wire could be traversed, by means of an external micrometer, across the centre of the tube, and remain parallel to the two flats. These flats were 1 cm. long and were separated by two semi-cylindrical surfaces of radius 0.4 cm.

Fig. 9 shows the velocity gradients across the small tube at Reynolds' numbers ranging from 15, at which the flow would presumably be stream-line, to 500.

§ 5. THEORY OF CONTINUOUS FLOW

For the equilibrium of a cylinder of fluid of length δl and radius r , we have

$$\pi r^2 \cdot \delta p / \delta l = 2\pi r l \cdot \tau,$$

where δp is the difference of pressure over the two ends of the cylinder, and τ is the frictional force on unit area of the sides of the cylinder.

In laminar flow $\tau = \eta dv / \delta r$, where η is the coefficient of viscosity. This leads to the familiar parabolic form of the velocity distribution across the cylinder when the condition of "no slip" at the boundary sides is introduced.

In turbulent flow, the form of the velocity gradient has been obtained from dimensional considerations by von Kármán*. Putting $y = r - a$, the distance out from the walls of a cylindrical tube,

$$v = f(\eta, \rho, \tau, y).$$

Imagine this to be expanded in powers of y , of which the first term is

$$v = f_1(\eta, \rho, \tau) y^q,$$

or, since $(\tau/\rho)^{\frac{1}{2}}$ and v/y have the dimensions of velocity, we can put

$$v = B \left(\frac{\tau}{\rho} \right)^{\frac{1+q}{2}} \cdot \left(\frac{v}{y} \right)^q \quad \dots\dots(4),$$

* *Abhand. Aero. Inst. Aachen*, 1, 6 (1921).

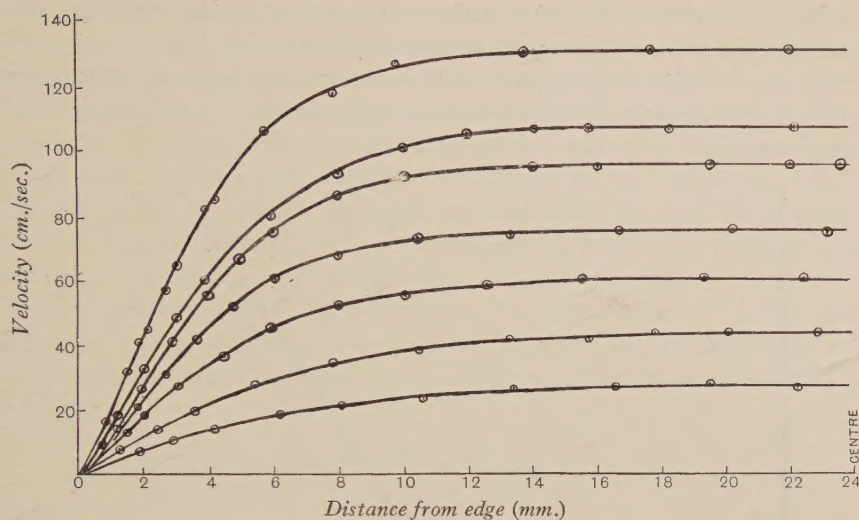


Fig. 8. One-way flow through square section pipe. Traverses 1 mm. inside opening.

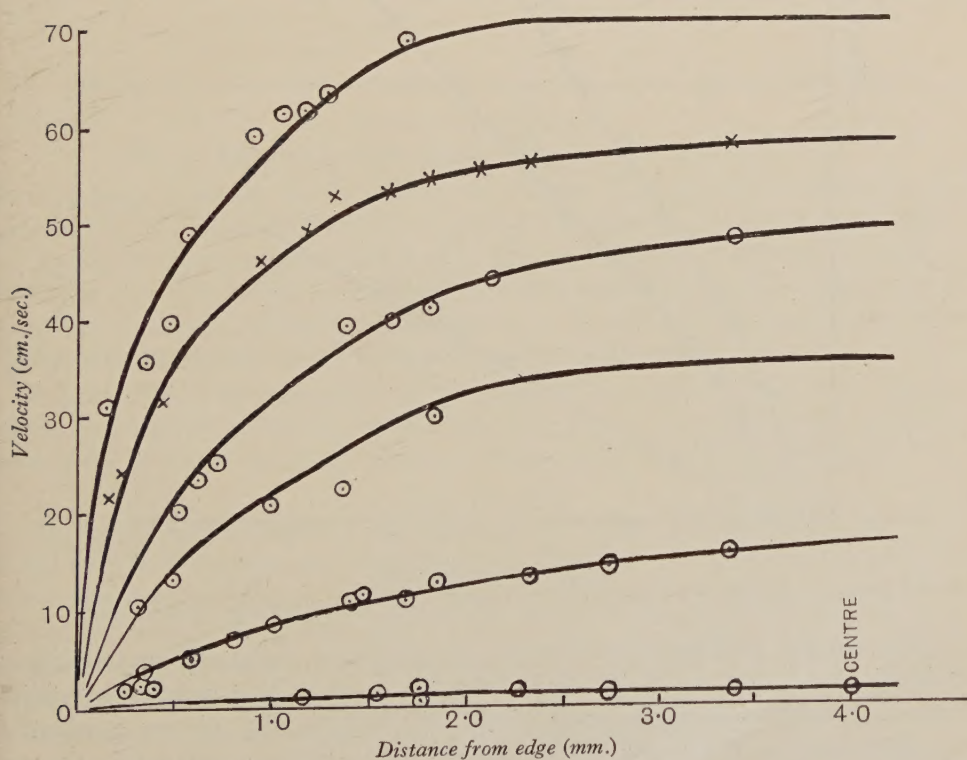


Fig. 9. Results for small oval pipe.

where B is a non-dimensional constant. To proceed further, we have to introduce the known variation of τ with v , or rather with outflow from a circular pipe. For this, von Kármán takes τ as proportional to $v^{7/4}$, whence $\frac{1}{2}(1+q) = 4/7$, and $q = 1/7$. This gives the so-called Prandtl law*, that in completely turbulent flow in circular pipes the seventh power of the velocity is proportional to the normal distance from the boundary.



Fig. 10. The Prandtl law for three different central velocities (large round pipe).

The velocity gradient at the boundary is, however, infinite from (4), giving an infinite value of $\tau = \eta \cdot dv/dy$. It is therefore assumed that laminar flow takes place in a region close to the walls of the tube wherein the velocity gradient is

* Apparently Prandtl had previously deduced this law but had not published it when von Kármán developed it further.

linear, merging into a gradient of the v^7 type in the body of the pipe. At high values of Reynolds' number the flow will more nearly approach the type $v = ay^{1/7}$ over the whole of the tube, or, alternatively, the laminar layer will get thinner, vanishing in the limit.

The first experiments on the velocity distribution in continuous flow in pipes were made by Stanton* and his collaborators by means of "disappearing Pitot tubes" of which one side of the orifice was formed by the wall of the pipe itself, the other side projecting to various small distances from the wall. These, calibrated in a known laminar flow, allowed measurements of velocity in turbulent flow to within 0.25 mm. from the wall of the pipe.

Nikuradse† traversed a Pitot tube across the end of pipes of various sections and about 2.5 cm. in diameter. Von Kármán has examined Stanton's results, and Nikuradse† his own results from the standpoint of the Prandtl law, and found

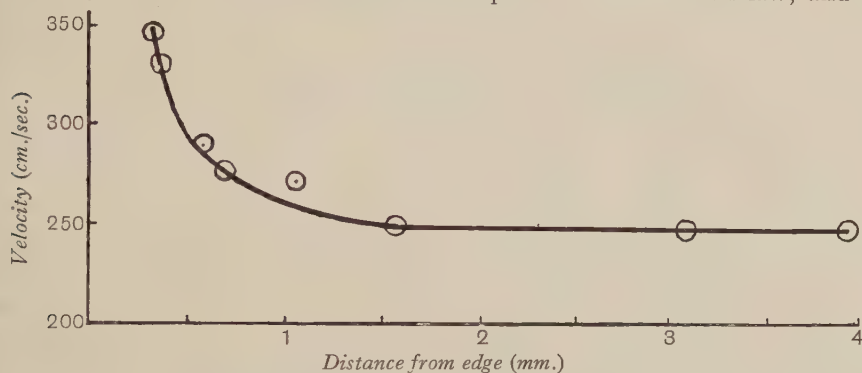


Fig. 11. Velocities at which stream became unsteady: small oval pipe.

good agreement outside the boundary layer (which Nikuradse's Pitot tube scarcely penetrated). We have tested our own results in the same way, by plotting v^7 against r for the higher average speeds in the pipe, Fig. 10. The best of these give a straight line from the centre of the pipe to within about 4 mm. of the walls, but within this boundary layer v is directly proportional to y . At intermediate speeds the actual distribution follows the fourth or fifth power of the velocity.

Incidentally, a determination of the pressure drop along a long pipe enables one to calculate τ and hence to find the numeric B , characteristic of turbulent flow, if the v^7 law be assumed. This was obtained by Nikuradse in the form

$$B^{-7/4} = 0.022.$$

By way of confirmation of the change over from stream-line to turbulent flow in the narrow oval pipe, we determined the velocity at which unsteadiness commenced by connecting the hot wire through a transformer to a string galvanometer, and taking records of the deflection of the string at various wind speeds. As long as the velocity was steady the string was undisturbed, but at a critical velocity oscillation of the string pointed to incipient turbulence. Fig. 11 shows the incidence

* Stanton, *Proc. R. S.* 85, 366 (1911). Stanton, Marshall and Bryant, *ibid.* 97, 413 (1920).

† *Forschungsarbeiten Verein. Deut. Ing. H.* 281. See also Döuch, *ibid.* H. 282, and Schiller and Kirsten, *Zeits. tech. Physik*, 10, 268 (1929).

of turbulence at various positions across the tube. It brings out well the fact that turbulence sets in over the whole of the tube at about the same velocity (250 cm. sec.), except within a millimetre or so of the walls, but that, as the mean speed goes up, this protective layer is reduced in thickness (to 0.3 mm. at 350 cm. sec.).

§ 6. ACKNOWLEDGMENTS

We wish to thank the Senate of the University of London for a grant from the Dixon Fund for the purchase of the Einthoven string galvanometer and the camera. Most of the practical work was carried out during 1928 at the College of Technology, Leicester, and we wish to acknowledge with gratitude the facilities which the authorities and Mr E. Langton, B.Sc., placed at our disposal in the Physics Department of that College.

DISCUSSION

Prof. Sir C. V. RAMAN said that when writing an article for the *Handbuch der Physik* he had been appalled by the difficulty of obtaining a direct method of treating the theory of musical instruments. Probably electrical analogies afforded the best approach to the problem, provided the results were subjected to correction by experimental observation. The annular region of high velocity which the author had found was so marked that it might conceivably be made visible by some suitable device. Had the author done any similar work on the flow of air through transverse holes? In the saxophone, for instance, where the finger holes are very large, the size of the holes might be important.

Mr C. M. WHITE: May I as a visitor express my thanks to the Society and to the authors of the paper for their account of a very interesting piece of work? My criticism of the paper—I feel that the authors have carried the desire for conciseness to a point at which it becomes almost a fault. I personally should have appreciated a more comprehensive account of the origin and aims of the work. I am in doubt also as to the extent to which the authors' aims have been realised. The apparatus too—experimental apparatus doesn't just happen—it is usually the result of careful design—a compromise between conflicting considerations—in their paper the authors give but little indication of the factors which led to the design of their apparatus. I for one should have found an account not only of interest but also of real value. I feel that there is this tendency towards undue briefness throughout the scientific writings of to-day, with the result that the reader is forced to narrow his field of interest. With regard to the results. The confirmation of the Rayleigh curves is most convincing and particularly valuable in view of the relative scarcity of sound theoretical treatments of the subject. Do the authors see any way to solve the problem of an alternating flow superimposed on continuous flow in long pipes? Data on this question would be of immediate value to engineers.

I am not yet convinced that the hot-wire method of measurement is sound quantitatively. In a sense, Fig. 4 is a proof of its unsoundness, for here the average velocity is zero, yet the wire shows definite cooling. I should expect a hot wire to

read high in a position where the flow is highly turbulent, and the upper curve of Fig. 8 seems to show evidence of this. I do not think the approximation to the seventh-power law shown in Fig. 10 is very convincing as a proof that the hot wire is reliable. This law is an approximation based on experiments with long smooth pipes. If the surface were rough, then the index would be 4, while a short streamlined orifice gives almost infinity as the index—the possible range therefore is very wide.

The phenomena described in connexion with Fig. 11 may be explained, I think, by the action of disturbances originally in the air as it enters the pipe—possibly these act in conjunction with the eddy-producing capabilities of the wire itself—but whether this is so or not the disturbance will only reach the wire at fairly high Reynolds' numbers—with small flows they will die out first. I think the term "critical velocity" has come to imply instability and is perhaps misleading in connexion with Fig. 11. Unsteadiness is present apparently long before the velocities of Fig. 11 are reached, as Fig. 9 shows considerable deviation from the Poiseuille distribution for all except the lowest two curves.

Dr W. N. BOND: The experiments by Carrière, mentioned in the second footnote of this paper, would be interesting if carried out for the space just outside the open end of the pipe, as the direction as well as the amplitude of the oscillations might be found. Departure from the v^7 law (Fig. 10), even at a distance from the pipe wall, was attributed by Mr White to the possible roughness of the pipe wall. I feel that the law is in any case not to be expected to hold accurately, as it may be affected by the distance from the entrance to the pipe, and also because it is based on the approximate assumption that $\tau \propto v^{7/4}$. I should like to ask the authors what were the characteristics of the beginning of turbulence as indicated by the string galvanometer. In some experiments in which I was able to hear the beginning of turbulence (for instance, using a stethoscope) it appeared to be quite random, and like a series of pistol shots. Finally, may I express my appreciation of the work the authors have brought before us?

Mr W. A. BENTON said that, while the authors had clearly established the existence of the annular region of high velocity, it was difficult to picture the reason for this, from the point of view of common sense. Could the authors give any summary explanation of it, and show where the energy in the region came from?

Prof. W. WILSON: A simple illustration may perhaps assist in removing the difficulty mentioned by the last speaker. Imagine a wide cylindrical vessel filled with a liquid and having a small orifice on one side near its base. I will suppose for the moment that the viscosity of the liquid is zero or negligibly small. I believe it is easy to show, with the aid of Bernoulli's theorem, that the velocity in the emerging jet at points in the plane of the orifice is greatest at the edge and has a minimum value somewhere near the middle. A little outside the orifice, in the *vena contracta*, the velocity will not vary throughout the cross-section of the jet but (assuming no viscosity) will have the Torricellian value $v = (2gh)^{\frac{1}{2}}$. Actually

the liquid has viscosity; but if this is not too great it will not seriously affect the general character of the velocity gradient in the plane of the orifice except near the edge where one would expect a steep gradient like those shown in the authors' curves.

Mr A. M. CASSIE: The peaks are not really difficult to explain. They are clearly analogous to the skin effect in an electrical conductor with the additional condition that the velocity must be zero at the walls of the tube. The skin effect in a conductor of circular cross-section, you will remember, can be shown to be the result of the variation of the mutual inductance between the elementary current cylinders with their radii. In the same way in the problem under discussion the force due to viscosity on an elementary cylinder varies with its radius. I find the two problems lead to the same Bessel equation, and making the velocity zero at the walls leads to the same shape of curve as that observed by Richardson and Tyler. The solution also shows that the sharpness of the peaks increases with the frequency and with the density of the gas while it decreases with the viscosity.

The PRESIDENT asked whether the authors had estimated the impedance of an aperture in an infinite plane wall or of the end of a pipe in an infinitely extended medium, utilising their results? The possible difference between the calibration of the hot-wire apparatus in turbulent and in stream-line flow had probably some bearing on the calculation of the impedance.

AUTHORS' reply: We have no data for the flow through side holes to put before Sir C. V. Raman, but some qualitative measurements at a side hole in a sounding flute have shown the same effect. The theory of wind instruments having side holes has been considered by one of us as an exercise in summed impedances in an appendix to a recent book, but it has not yet been possible to work out the impedance of an orifice from theory, taking into account the annular effect, as suggested by the President.

We are inclined to agree with Mr White that the paper is somewhat compressed—at least as regards data—two of our graphs having been rejected as superfluous! As regards exposition, he must remember that it is a second paper on the subject, and we hope he will find the *raison d'être* of the second paper in the first. The points raised with regard to alternating plus continuous flow (which we hope to find time to investigate) and the use of a hot wire in a turbulent stream are very relevant. To the extent that turbulent motion can be regarded as continuous flow on which an irregular ripple has been superposed, the hot wire will indicate a slightly higher average (towards the peaks of the ripples) than in a steady draught. However, calibrations in a wind channel and on a whirling arm tally to an extent which seems to indicate that the difference is a small one. Vorticity would not be produced by the wire itself at speeds below 2000 cm. sec. Mr White views Fig. 10 in a novel light. We were not trying to test our hot-wire calibration by the v^7 law, but boldly assumed that our calibration was correct, and subjected

the v^7 law to the test in Fig. 10. We agree with Dr Bond and other speakers that the agreement may be deceptive; a v^6 or a v^5 law would look almost equally well on the graph, but we wanted to test the Prandtl-Kármán law, which, as Dr Bond remarks, is based on an assumption as to the resistance of pipes which has been only approximately verified in practice. Some analogies between a.c. electricity and ideal orifice flow were considered in the first paper and the discussion thereon. Mr Cassie's mathematical solution, which we have seen, is a neat alternative to that given in the paper. Finally, we would thank all speakers for their kind interest.

RESOLVING POWER TESTS ON MICROSCOPE OBJECTIVES USED WITH ULTRA-VIOLET RADIATION

By B. K. JOHNSON, F.R.M.S., Demonstrator in the Technical Optics Department, Imperial College of Science and Technology.

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ABSTRACT. The work described was conducted with the view of determining the extent to which the increased resolving power which should be obtainable by the use of ultra-violet radiation instead of visible light is realisable with modern microscope object-glasses, computed for use with ultra-violet radiation. The method, which provides a quantitative rather than a qualitative test on an objective, was first developed for visual lenses and has now been adapted for use with ultra-violet objectives. An object of known regular structure and of variable interval is produced by projecting in the object-plane of the lens under test a reduced image of a grating, the apparent line separation of which is varied by rotation of the latter; thus the line interval can be determined when resolution just ceases. The results show that the fused quartz monochromatic object-glass of numerical aperture 0.35, computed for and used with radiation of wave-length 0.275μ , gives nearly twice the resolving power of a lens of similar aperture computed for and used with light of wave-length 0.51μ ; while the fused quartz monochromatic lens of numerical aperture 1.2 has a resolving power 70 per cent. higher than that of a well-corrected object-glass of the same numerical aperture when used with light of wave-length 0.51μ .

§ 1. PURPOSE OF THE WORK

IN a paper* read before the Royal Microscopical Society in April 1928, the writer described a method whereby a numerical value for the optical resolving power of microscope objectives could be obtained. Full details of the method will be found in that paper, but briefly it consisted in forming a reduced image of a relatively coarse grating either by an auxiliary microscope objective or by the lens itself (i.e. with a vertical illuminator), and using this image as the object for the lens under test. By rotation of the grating the apparent distance between the lines of the object is varied and a position found when resolution just ceases, from which the line separation is obtained.

As it was possible by this means to obtain a quantitative rather than a qualitative test (such as is given by observations on natural objects) on the performance of a lens, the method was further developed for use with the quartz monochromat series of lenses computed for a wave-length in the ultra-violet region of the spectrum ($\lambda = 0.275\mu$), the aim being to determine how much increased resolution really was being obtained by the use of a shorter wave-length.

* *Journ. Roy. Micro. Soc.* **48**, 144-158 (1928).

§ 2. THEORY OF THE METHOD EMPLOYED

The formation of a reduced image produced by a lens system when the object is rotated will be readily remembered by reference to Fig. 1. Lateral and axial magnifications being proportional to M and M^2 respectively, it follows that the axial length l' of the reduced image $x'y'$ will be equal to l/M^2 , and therefore will be distinctly small in comparison with l . Provided l' be less than the depth of focus of the lens to be tested, the image $x'y'$ acts as a sensibly flat object, and the spacing of the lines of this imaged object will be proportional to the cosine of the angular swing of the grating xy .

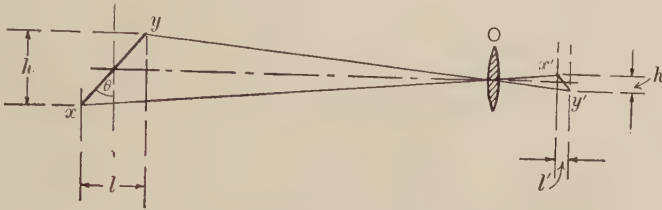


Fig. 1. Image of inclined object.

The depth of focus in the object space of an objective being equal to

$$\lambda/4n \sin^2 (U/2)^*$$

(where U is the angular semi-aperture of the cone of rays entering the lens), it will be seen that the requirement that l' shall not exceed this depth of focus imposes a necessary limit on the maximum permissible rotation of the grating; therefore it is desirable to choose a grating such that resolution will cease (for the lens under test) before the grating reaches this maximum angular position. Also it is necessary to limit the field in which the imaged grating lines are formed: this is done by means of a line ruled on a silvered cover glass or alternatively by limitation of the width of the grating proper.

Example. In the case of the two ultra-violet lenses used in these tests, where the imaged object is formed by a 2 mm. immersion monochromat; if the reduction is 90 times, the length xy of the portion of the grating used is 1.7 mm. and if it is rotated through 60° for example, then l' will be found to be equal to 0.00018 mm. or 0.18μ .

If a 6 mm. monochromat were to be used to view this image, then its depth of focus $= \lambda/4n \sin^2 (U/2) = 2.2\mu$. Thus l' is well within the permissible value for producing a flat object for the 6 mm. lens under test. If the lens under test is a 2 mm. monochromat, its depth of focus is 0.2μ and therefore the imaged object as before will still serve for testing the last named lens.

On this principle, therefore, from a knowledge of the size of the grating space in the imaged object when the grating is normal to the optical axis, and by determination of the angle at which resolution of the lines just ceases, it is possible to obtain a numerical value for the resolving power of a given objective, for if d is

* See section on "Optics of the Microscope" (Prof. A. E. Conrady) in *Dict. Applied Physics*, 4, 218-223.

the size of a grating space of the reduced image when the grating is normal, and if θ is the angular turn of the grating when resolution ceases, then

$$d_{\theta} = d \cos \theta.$$

A verification of the validity of this simple formula was obtained when the tests were made with objectives used in visual light; a full discussion of the theory, together with experimental confirmation, was given in the paper first referred to above, and need not be repeated here. As a matter of interest, however, a summary of the resolving-power values, as determined by the method for three visual objectives is given in the table.

Table 1. Resolving powers of three visual objectives.

Objective (nominal)	Numerical aperture	λ	Observed resolution, mm.	Theoretical resolution mm.
1 in.	0.28	0.51 μ	0.00094	0.00091
1 in. oil	0.71	0.51 μ	0.00039	0.00036
$\frac{1}{2}$ in. oil	1.2	0.51 μ	0.00024	0.00021

§ 3. EXPERIMENTAL WORK

Owing to the exacting character of the practical work with the ultra-violet lenses it is thought that some details of this may be worthy of description. The apparatus took the form previously described by Dr L. C. Martin and the author*, the microscope stand being that designed by Mr J. E. Barnard. Modifications

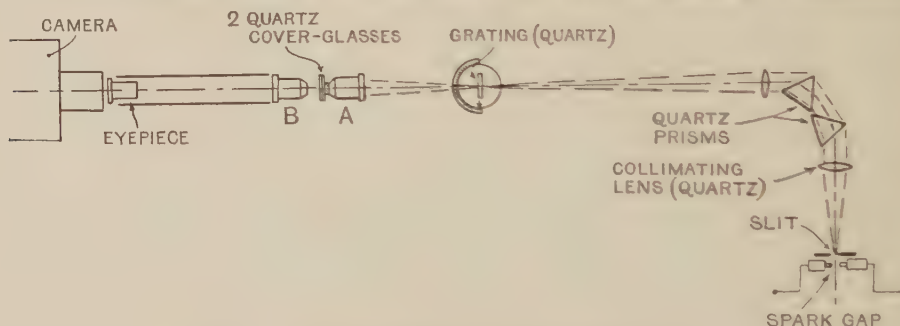


Fig. 2. Schematic diagram of the optical system.

to meet the requirements of the tests had to be made, and Fig. 2 shows diagrammatically the arrangement of the components. All the optical parts were of quartz. The fittings shown on the right of the figure constitute a monochromator by means of which the light from a high potential spark, of the order of 10,000 volts, formed between cadmium electrodes is dispersed sufficiently to bring one line (namely the 0.275μ line) of the cadmium spectrum on to the quartz grating. An image of the latter† is formed by the objective *A* and this image is in turn observed by the lens *B* under test.

* "Ultra-violet Microscopy," *Journ. Sci. Inst.* 5, 337-344, 380-386 (1928).

† It will be noted that this is an *enlarged* image.

§ 4. TESTING OF 6 MM. MONOCHROMAT

For the test on the low power quartz lens, the 2 mm. Beck immersion monochromat was used as the object-forming lens, and the microscope stand was reversed, see Fig. 3; the purpose of this arrangement was to make use of the specially constructed fine adjustment (reading to 0.1μ) for the high power lens, and of the somewhat coarser adjustment (provided for the condenser) as the focussing movement for the 6 mm. objective, for which the depth-of-focus requirements are not so exacting as for the former lens. The figure illustrates the way in which the sub-stage condenser mount has been converted into the microscope proper, the tube *M* (carrying the quartz eyepiece) fitting conveniently into the



Fig. 3. Reversal of microscope for 6 mm. object glass test.

mount. The immersion objective *O* is also seen, whilst the grating *G* with its divided circle is held in a suitable fitting in the position usually occupied by the eyepiece. A quartz slide was specially prepared: this consisted of two fused quartz cover glasses (one of which was silvered and had a ruled line 0.008 mm. wide) cemented together with a thin film of glycerin between them; the line served as the object on which the 6 mm. lens was focussed. Focussing was facilitated by illumination of the object in the way already described; the line was focussed roughly with the fluorescent eyepiece and a series of photographs was then taken to obtain the exact focus. Without further alteration to this end of the microscope stand, the grating was placed in position (*G*, Fig. 3) at the correct tube-length for the 2 mm. monochromat, the latter being already in place and acting as a condenser for the first part of the experiment. By observation through the fluorescent eyepiece, the 2 mm. lens could now be moved by means of its own fine adjustment until a rough outline of the grating was seen approximately in focus with the line

object, and then a further series of photographs was taken, the focus of this lens being varied by 0.2μ each time until the precise focus of the grating lines was found.

The line on the silver film and the imaged lines of the grating having been obtained simultaneously in focus on the plate*, the grating was then rotated through successive angular amounts and a series of some twenty exposures were

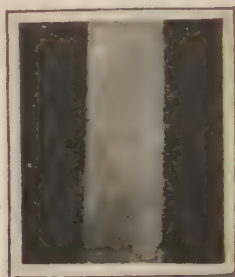


Fig. 4. 6 mm. monochromat under test.
Grating normal. $\lambda = 0.275\mu$. $M = 1700$.

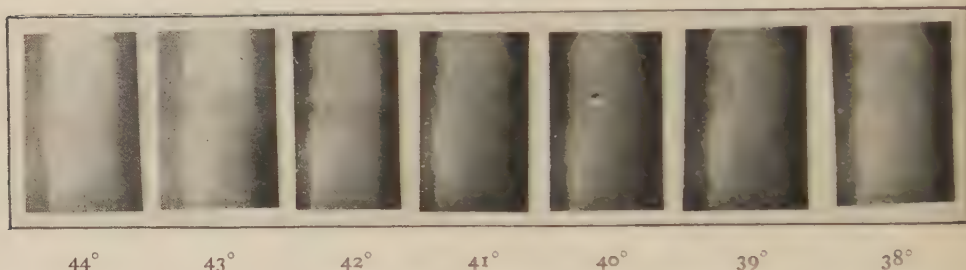


Fig. 5. Successive rotations of grating. 6 mm. monochromat.
Resolution ceases at 42° . $M = 1250$.

taken on a film as before. On development of this the angular position of the grating at which resolution ceased could be found, and the object-interval determined. Fig. 5 shows seven successive exposures of the series thus taken. It is feared that the photomicrographs are not very suitable for reproduction. The originals will be gladly shown to anyone who is interested.

Details of the actual figures obtained with this lens are given below:

6 mm. quartz monochromat.

Numerical aperture: 0.35 .

Wave-length: 0.275μ (Cd spark).

Object-forming lens: 2 mm. quartz monochromat.

Grating employed: 20 lines to the mm. ruled on a silvered quartz plate.

* Fig. 4 shows the actual photomicrograph of this.

With the grating normal to the optical axis, the line interval of the imaged object, determined photographically, was 0.00053 mm.

Photographs were taken at 32°, 33°, 34°, 35°, 36°, 37°, 38°, 39°, 40°, 41°, 42°, 43°, 44°, 45°, 46°, 47°, 48° and 49°. Resolution ceased at 42°.

∴ line interval $d = 0.00053 \times \cos 42^\circ = 0.00039_3$ mm.

By check method.

Width of line on silver film: 0.008 mm.

Number of lines immediately before resolution ceases: 20.

∴ d (by check) = 0.00040 mm.

Theoretical resolution: 0.00039₁ mm.

The result indicates clearly the advantage of using the shorter wave-length, as the resolving power of this objective is the same as that which it is possible to obtain with an objective of double the numerical aperture (namely a $\frac{1}{6}$ in. object-glass of numerical aperture 0.71), used with light of wave-length 0.51 μ .

§ 5. SOME EXPERIMENTAL DETAILS

Mention must here be made of certain details, only briefly alluded to in the foregoing description of the method, but such that without them it would not have been possible to carry out the tests.

First of these, was the use of the stable immersion fluid for the 2 mm. monochromat. A full description of this glycerin and cane-sugar solution is given in the *Journal of Scientific Instruments*, in the article already referred to, and the test was made possible by the fact of being able to use this fluid for long periods—several hours—without the occurrence of any change in its refractive index. Obviously, considerable time is taken both in focussing with each objective in turn and also in taking a series of grating rotations, all three operations being photographic ones; and to have used the originally intended immersion fluid, namely glycerin and water, would have added to the difficulties very greatly, its hygroscopic nature (affecting its refractive index) produces a continual change in focus of the image.

Secondly, the preparation and making of the gratings and object-slide may be of interest. The former were ruled on silvered quartz slides by an attachment fitted to the geometric slide of a measuring microscope illustrated in Fig. 5 of the paper referred to in the footnote on page 16. With this three gratings were made, namely: (a) 20 lines per mm., grating space 0.05 mm.; (b) 40 lines per mm., grating space 0.025 mm.; (c) 67 lines per mm., grating space 0.015 mm. The first was ruled with a ground steel tool, whilst a carborundum chip, suitably mounted, was employed for the ruling of the latter two. A splinter from a carborundum crystal can be made to produce extremely fine and clean lines on silvered surfaces, and it was with such a splinter that the line 0.008 mm. wide was ruled for the object-slide. The use of a platinised surface aids in the obtaining of a clean cut, and it was found possible to rule lines with a carborundum splinter on such a surface of 0.002 mm. in width; further a platinised surface, produced by the cathode process, is more opaque

than silver to wave-length 0.275μ and is therefore better from this point of view also. Grating (*c*) was ruled on such a surface.

Thirdly, the best tube-length for each objective was determined by means of ultra-violet photomicrographs of the minute holes present in a chemically deposited silver film. If a series of photographs for a number of separate tube-lengths be taken, one will be found where the extra-focal images are similar in appearance, and this represents the best tube-length.

§ 6. TESTING OF 2 MM. MONOCHROMAT

The general outline of the method for testing the immersion quartz objective is similar. In this case, however, another immersion monochromat had to be used



Fig. 6. Apparatus when testing 2 mm. quartz monochromat.

for the object-forming lens and therefore it was necessary to obtain a lens which was at least as good in performance as the one under test. Consequently, a qualitative test was first carried out by the taking of comparison photomicrographs of a silver-film object with each objective* in turn; the results obtained showed that both the definition and resolution of the silver granules produced by each of the lenses were practically identical.

The arrangement of the apparatus for the resolving power test is seen in Fig. 6. As the objectives were both immersion lenses it was not necessary to have an object slide on the microscope stage; the space between the two front lenses of each objective was filled with immersion fluid. This was effected by means of a suitable bath formed by a flexible casing of thin rubber attached to the metal work

* One lens was the 2 mm. Beck monochromat and the other a Zeiss 2 mm. monochromat.

of each lens (see Fig. 7). Further, as no object slide was employed, it was necessary only to focus with one lens, namely that which was attached to the special fine adjustment movement of the microscope, whilst the other lens remained stationary in the sub-stage condenser mount.

The grating was set up as before (see Fig. 2) normal to the optical axis and at the correct tube-length distance for the object-forming lens; it was then illuminated with the 0.275μ Cd line and by use of the fluorescent eyepiece it was possible to focus the objective under test sufficiently well for the image of the outline of the grating to be seen, although no lines were visible. The camera was placed in position and a series of photographs was taken, the focus being varied by 0.2μ each time: the best focus was thus determined and the lines were clearly resolved on the plate. The grating was then rotated and a further series of photographs was taken

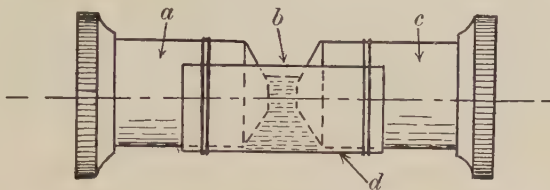


Fig. 7. Method of supporting immersion liquid between the two monochromats.

- | | |
|-----------------------------|---------------------------------|
| <i>a</i> 2 mm. monochromat. | <i>b</i> Immersion liquid bath. |
| <i>c</i> 2 mm. monochromat. | <i>d</i> Thin rubber casing. |

for successive increases in its angular position, the point being found when resolution ceased. When the grating was near this position it was felt desirable to make perfectly sure that the lines were still precisely in focus, on account of the author's previous experience with such a test when using a $\frac{1}{12}$ in. oil immersion lens in visual light, and therefore for each orientation of the grating from then onwards a series of five photographs was taken, the focus being altered each time in order to secure the best setting. An illustration of this is given in Figs. 8, 8(*a*), 8(*b*), where it will be seen that the centre photograph in each case is in focus, whilst the two outer ones are not. By taking this precaution one felt more confident that a wrong resolution value due to incorrect focussing of the grating lines would be obviated.

Two entirely independent measurements by the method were made, one with a grating of 0.025 mm. interval, and the other with one of 0.015 mm. line space, the limit of resolution in each case lying between 0.00012 mm. and 0.00013 mm.

2 mm. quartz monochromat.

Numerical aperture: 1.2.

Wave-length: 0.275μ (Cd spark).

Grating—interval: 0.025 mm.

Magnification of microscope proper, determined photographically: 840.

With the grating normal to the axis the line interval of the imaged object was 0.00027_2 mm.

When the grating was rotated to 60° the line interval d was $0.00027 \times \cos 60^\circ$ or 0.000135 mm. The lines were still resolved.

With the grating at 63° , $d = 0.00027 \cos 63^\circ = 0.000122$ mm., the lines being no longer resolved.

By check method.

Grating—interval: 0.015 mm.

Magnification: 840 .

With the grating normal to the axis, the line interval of the imaged object was 0.000171 mm.

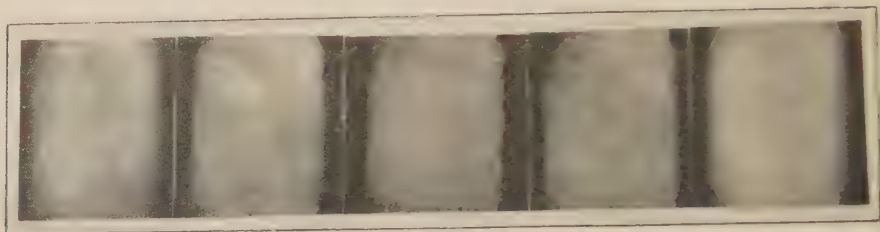


Fig. 8 (a). Object interval 0.000135 mm. Clearly resolved. 2 mm. monochromat.
 $\lambda = 0.275 \mu$. $M = 3260$.



Fig. 8 (b). Object interval 0.000124 mm. Not resolved. 2 mm. monochromat.
 $\lambda = 0.275 \mu$. $M = 2600$.

With the grating rotated to 38° , the line interval was $0.00017 \times \cos 38^\circ$ or 0.000134 mm. This was still resolved.

With the grating rotated to 43° , $d = 0.00017 \times \cos 43^\circ = 0.000124$ mm., the lines being no longer resolved.

Theoretical resolution: $d = 0.000114$ mm.

This test, however, where the two lenses are of almost the same numerical aperture, may be looked upon as a test of the two lenses combined, for the limit of resolution as measured will be that given by the poorer objective, and thus a possible objection to the method might be raised. If, for instance, the image-forming lens had errors which were just compensated by errors in the lens under test, or conversely were not corrected by corresponding errors of the latter, in either case an incorrect value for the resolving power of the lens under test would be obtained. This objection can be met when it is possible to use a good objective of higher numerical aperture as the object-forming lens. For this reason it was hoped to employ the vertical illuminator method, depicted in Fig. 9, which was

used so successfully with visual objectives. For although, on account of the light's travelling twice through the lens, all aberrations would be doubled, with a consequent lowering effect on the resolving power, nevertheless the vertical illuminator method has the merit of providing a relative test for objectives of the highest possible numerical aperture. Incidentally it has the additional advantage of using only the one object-glass instead of two.

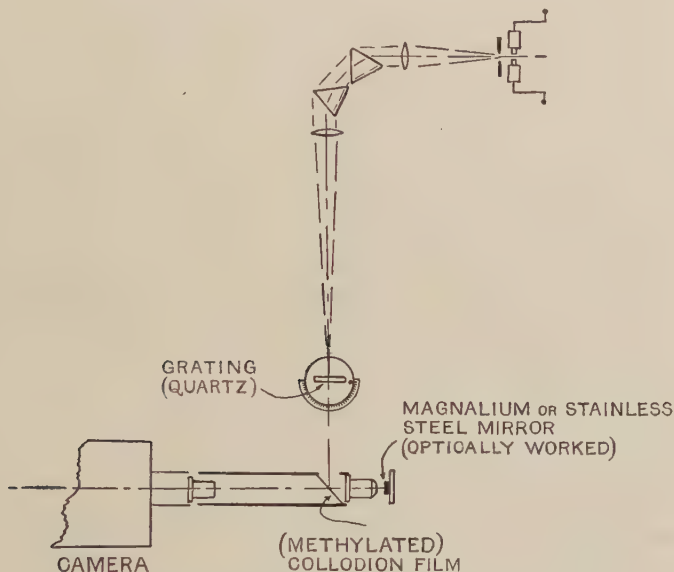


Fig. 9. Diagram of vertical illuminator method.

Although this form of the test was carried out quite satisfactorily for a 2 mm. immersion lens in visible light, it was not found possible to do this for either of the two available high-power ultra-violet objectives, the reason being that the design of the component lenses of their systems is such that an unusually large percentage of the incident radiation is reflected back along the axis of the microscope, thus making it almost impossible to obtain satisfactory photographs.

Hopes had also been entertained of carrying out a further test, by the use of finely ruled gratings on quartz and by direct observation of these with the 2 mm. monochromat, but it has not yet been possible to obtain a satisfactory ruling of the necessary fineness.

In any case, the test as it was made reveals the fact that the two lenses together will resolve two objects separated by a distance of 0.135μ , and shows therefore that a distinct advantage is being obtained over a $\frac{1}{12}$ in. objective, used in visible light, whose resolving power is 0.21μ .

§ 7. CONCLUSIONS

The work carried out indicates that the method can be applied for determining a numerical value for the resolving power of objectives used with ultra-violet radiation. It further illustrates that the use of shorter wave-lengths with micro-

spectro objectives is justified, inasmuch as it increases the resolution obtainable, although the difficulties involved both in the manufacture and in the manipulation of the apparatus are at present admittedly great. It will be noticed that the 6 mm. monochromat gives almost its theoretical resolution, but the 2 mm. does not go so near to that limit; nevertheless the fact remains that something of the order of a 70 per cent. increase in resolving power has been obtained over what is physically possible with the highest-power objective in visible light, and therefore it is felt that in this respect at least a step forward has most certainly been made.

DISCUSSION

Mr T. SMITH. There is one point of some importance on which this paper is silent—the filling of the effective part of the grating by the image of the spectroscope slit formed by light of the wave-length used. I suggest the author should state the conditions secured in his experiments when the paper appears in the *Proceedings*. As there is still a good deal of difference of opinion among microscopists on microscope theory—and the inclusion of physicists would widen the field—the results Mr Johnson has obtained for the ultra-violet resolution with quartz objectives should be considered in conjunction with the corresponding visual tests given in Table 1. The observed values in both cases are reasonably near the corresponding theoretical resolutions obtained by using the same empirical factor. Exact comparison is not possible for a number of reasons, including the use of a wide band of wave-lengths in one case and a single line in the other. The conclusion, however, that the objectives are good enough to yield approximately the increase in resolving power that might be hoped for by the use of ultra-violet light is justified, and this result should encourage the makers in any further developments of such objectives. Mr Johnson is to be congratulated on his success in making such very difficult measurements.

Mr W. W. BARKAS. I should like to ask whether the increased resolving power obtained by the use of ultra-violet light would make it possible to obtain real images of colloidal particles with a view to measuring their radii directly. It is really a question of the time of exposure required since in liquid media the Brownian motion is fairly rapid.

Dr L. C. MARTIN (communicated). A quantitative test on the resolving power of ultra-violet microscope objectives of high numerical aperture has hitherto offered the greatest difficulties. Efforts have been made to produce test rulings of the order of 200,000 lines to the inch, but we have not been able, so far, to obtain a satisfactory object of this kind to test the resolving power. Moreover it is somewhat difficult with a grating of the ordinary type to determine the optimum performance of the objective, which may have to be stopped down in order to secure sufficient contrast in the image. Mr Johnson's "grating" with its transparent and opaque spaces lends itself more readily to experiments at full aperture. The deficiency in the actual as compared with the theoretical performance of the

objectives is almost certainly due, in my opinion, to the residual lack of homogeneity of the fused quartz. It is an open question whether, in view of all the increasing difficulties of manipulation, and this lack of homogeneity, it would pay, at present, to try to construct objectives computed for use with wave-lengths shorter than $\lambda = 0.275\mu$, but if it is tried then Mr Johnson's test will give ready information as to the result.

AUTHOR'S reply. In reply to the point raised by Mr Smith as to the method of filling the effective part of the grating with the desired wave-length, I fear that I did not originally make it sufficiently clear—although the monochromator in Fig. 2 partly suggests it—that an *enlarged* image of the 275 Cd line was formed in the plane of the grating. Its size is proportional to the ratio of the focal length of the telescope lens to that of the collimator lens used in the monochromator, which was in this case about 3. As the actual width of the enlarged image was approximately 6 mm. and the necessary width of the grating 2 mm., the total effective area of the grating was always filled with the required wave-length. I would like to thank Mr Smith for his appreciative remarks concerning the difficulties of the work. Colloidal particles of the size referred to by Mr Barkas are far below the capabilities of resolution of even the ultra-violet microscope. As is well known, the light scattered by such particles may be rendered visible by the use of the modern high-power dark-ground illuminator employing visible light, but it is nevertheless practically impossible to take satisfactory photomicrographs of these owing to the Brownian movement. In the case of the ultra-violet microscope the only way of recording a precise image is by photography, and as the length of exposure required for dark-ground ultra-violet illumination is at present far from “instantaneous”, I am afraid there is little hope of obtaining the information (such as the diameter of these particles) in which Mr Barkas is interested, even with this microscope.

SOME ADDITIONAL LINES IN THE SECONDARY SPECTRUM OF HYDROGEN

F. C. CONNELLY, A.R.C.S., B.Sc., D.I.C.

Communicated by Prof. A. Fowler, F.R.S., June 22, 1929.

Taken as read, October 25, 1929.

ABSTRACT. The author had measured about 1000 lines in the hydrogen spectrum between λ 4735 and λ 4018 when the similar work of Gale, Monk and Lee was published in 1928. The author's list included, however, about 280 lines not given in the prior list, and these are published in the present paper.

§ 1. INTRODUCTION

BEFORE the publication in March 1928 of Gale, Monk and Lee's "Wave-lengths in the Secondary Spectrum of Hydrogen*," the usual standards of reference used in attempts to analyse the spectrum had been the tables published by Merton and Barratt† some seven years before. These tables, however, were not complete, as a large number of fainter lines were not measured and numerous brighter ones were not completely resolved into their components. Tanaka‡ and Deodhar§ filled in some of these gaps, but do not appear to have used standard iron lines for calibration purposes, having based their measurements on some of the stronger hydrogen lines measured by Merton. These additional tables are therefore less accurate than the original ones. The dispersion used by Merton, 10 Å. per mm., was quite moderate so that the limit of resolution appears to be 0.3 Å. From attempts which have been made to select bands in the spectrum, this lack of resolution seemed a serious defect, and it was with the object of clearing up the difficulties in the blue and violet that a remeasurement of these regions was commenced early in 1927.

In all about 1000 lines were measured between 4735 Å. and 4018 Å. Many of those given by Merton as single were found to be double or even triple and numerous new lines were measured. The publication of these values was delayed pending further experiments, and in the meantime the tables by Gale, Monk and Lee, mentioned above, have appeared and have been accepted by most workers as satisfactory standards. These values have been compared carefully by the present author with his own values and in nearly all cases the agreement has been

* *Astrophys. Journal*, **67**, 89 (1928).

† *Proc. R. S. A.*, **113**, 420 (1926).

‡ *Phil. Trans. A*, **222**, 69 (1921).

§ *Proc. R. S. A.*, **108**, 592 (1925).

very good. No useful purpose would therefore be served by publishing the complete tables. As, however, a number of faint lines are not given in Gale, Monk and Lee's tables, it has been felt that these additional lines may be found valuable in extending bands already discovered and they are accordingly given below in Table 1.

A few lines measured by Gale, Monk and Lee, and given in their tables, have not been found by the author. These lines are printed below in Table 2.

§ 2. EXPERIMENTAL METHOD

The source of the spectrum in these determinations has been an H-tube of the usual type having a capillary 8 cm. long and 1 mm. in diameter. The type of tube used by Wood*, and designed by him to give the secondary spectrum at its strongest, has also been used, but the extra complication involved in water-cooling the electrodes, a precaution which was necessary when 2 or 3 kw. were to be dissipated in the tube, greatly offset the advantages of shorter exposure and freedom from the Balmer series attainable with the special tubes. Indeed the presence of the line spectrum was not disadvantageous, nor was the exposure unduly long when the H-tubes were used, so that the latter were generally adopted.

The effect of pressure on the spectrum is appreciable, the optimum pressure with these tubes being 2 or 3 mm. of mercury. This appears to be higher than the pressure used by Gale, working with a larger tube similar to that of Wood.

With a 10 ft. concave grating a dispersion of 2.5 Å. per mm. was obtained in the second order, and an exposure of 2 hours was sufficient to give a strong photograph on Imperial ordinary plates (H and D 90). The measurements were not, however, solely dependent on the grating plates as a two-prism Littrow instrument was used for checking many of the lines. This instrument was specially arranged for the purpose and gave a dispersion of 1 Å. per mm. at 4000 Å. and 2 Å. per mm. at 4400 Å. It was slower than the grating, and owing to slight optical defects of the prisms a slight blurring of the stronger lines could not be prevented from becoming apparent under high dispersion. This defect prevented the resolution from being any better than that yielded by the grating.

§ 3. DISCUSSION OF RESULTS

In the original tables drawn up by the author, wave-lengths were given to 2 or 3 decimal figures according to their probable accuracy. As the additional lines given below are the faintest ones, capable of least accurate measurement, most of the values are given to 2 decimal places only.

The line given as 4450.346 Å. by Gale, Monk and Lee is presumably a misprint for 4451.346 Å., this being in good agreement with the author's value 4451.35. Where a line has previously been measured by Deodhar or Tanaka, the fact is indicated by D or T against it.

* *Proc. R. S. A.*, 97, 455 (1920); *Phil. Mag.*, 6, 251, 729 (1921).

§ 4. NOTATION

The system of notation used in this work is that adopted in the "Revision of Rowland's *Preliminary Table of Solar Spectrum Wavelengths*" (1928), the faint lines being denoted by 0, - 1, - 2, - 3.

§ 5. ACKNOWLEDGMENT

In conclusion the author wishes to express his thanks to Prof. A. Fowler for his continued interest and assistance throughout these measurements.

NOTE ADDED OCTOBER 1929

During the preparation of this paper further wave-lengths were published by Finkelburg*. As, however, many lines remained unmeasured, it was decided not to recast the paper.

* *Zeits. für Physik*, 52, 27 (1928).

Table 1. Lines measured by the author and not given by Gale, Monk and Lee.

Wave-length	Wave-number <i>in vacuo</i>	Wave-length	Wave-number <i>in vacuo</i>
4725.23 (-3)	21157.08	4540.66 (-2)	22017.08
4703.88 (-3) D	21253.11	4539.60 (-2)	22022.22
4696.31 (-3)	21287.38	4538.67 (-1)	22026.73
4695.70 (-3)	21290.14	4536.87 (-3)	22035.46
4695.12 (-3)	21292.77	4536.40 (-2)	22037.75
4694.78 (-3)	21294.31	4535.12 (-1)	22043.98
4682.59 (-2)	21349.95	4532.18 (-3)	22058.27
4676.73 (-1)	21376.50	4531.64 (-2)	22060.90
4668.573 (2)	21413.84	4530.53 (-1)	22066.30
4664.82 (0)	21431.07	4528.75 (-2)	22074.98
4654.76 (-1)	21477.39	4524.72 (-3)	22094.63
4646.94 (-3)	21513.53	4524.50 (-3)	22095.71
4646.40 (-2)	21516.04	4522.09 (-3)	22107.48
4644.19 (-1)	21526.27	4521.80 (-3)	22108.91
4637.68 (-1) D	21556.49	4521.10 (-3)	22112.33
4635.98 (-3)	21564.40	4498.95 (-3)	22221.19
4630.95 (-3)	21587.81	4494.49 (-3)	22243.25
4623.26 (-1) T	21623.71	4494.19 (-2)	22244.73
4622.73 (-2)	21626.20	4492.21 (-3)	22254.53
4621.15 (-3)	21633.59	4491.29 (-2)	22259.09
4619.09 (-3)	21643.24	4491.12 (-2)	22259.93
4615.64 (-3)	21659.43	4490.62 (-1)	22262.42
4615.21 (-3)	21661.44	4489.17 (-1)	22269.50
4610.76 (-3) D	21682.35	4488.96 (-1)	22270.64
4610.07 (-3)	21685.59	4480.40 (-1)	22313.19
4606.53 (-3) D	21702.25	4478.38 (-1) <i>d</i>	22323.26
4601.37 (-2)	21726.59	4477.62 (-3)	22327.05
4595.54 (-3)	21754.16	4476.72 (-3)	22331.54
4592.21 (-1)	21769.94	4473.42 (-3)	22348.01
4591.47 (-1)	21773.43	4472.86 (-2)	22350.80
4590.21 (-1)	21779.42	4468.91 (-2)	22370.56
4588.08 (-3)	21789.53	4468.55 (-3)	22372.37
4586.96 (-1)	21794.84	4468.11 (-2)	22374.57
4586.23 (-3) T	21798.31	4466.23 (-2)	22383.98
4586.00 (-2)	21799.40	4462.93 (-2)	22400.54
4585.60 (-2)	21801.30	4462.53 (-3)	22402.55
4580.47 (-1)	21825.73	4458.01 (-2)	22425.29
4576.94 (0)	21842.56	4454.34 (-1) D	22443.74
4573.34 (-3)	21859.75	4449.38 (-2) T	22468.75
4570.40 (-2)	21873.81	4442.07 (-2)	22505.73
4569.18 (-3)	21879.65	4441.11 (-1)	22510.58
4565.10 (-2)	21899.21	4440.12 (-3)	22515.61
4563.14 (-3)	21908.61	4436.81 (-3)	22532.40
4561.01 (-2)	21918.85	4434.09 (-1)	22546.23
4559.13 (-1)	21927.88	4433.72 (-3)	22548.10
4556.74 (-2)	21939.39	4433.41 (-1)	22549.68
4556.49 (-1)	21940.59	4433.05 (-2) D	22551.52
4555.32 (-3)	21946.23	4432.03 (-3)	22556.71
4552.72 (-1)	21958.76	4431.82 (-3)	22557.78
4552.65 (0)	21959.09	4429.90 (-3)	22567.56
4548.93 (-2)	21977.05	4429.56 (0)	22569.29
4545.48 (-2) D	21993.75	4428.23 (-3)	22576.06
4545.19 (-2)	21995.13	4425.35 (-3)	22590.76
4544.85 (-2)	21996.78	4424.93 (-2)	22592.90
4544.163 (1)	22000.11	4423.68 (-2)	22599.29
4542.17 (-2)	22009.77	4421.57 (-1)	22610.07
4541.62 (-2)	22012.44	4420.71 (-3)	22614.47

D, T: previously measured by Deodhar and Tanaka respectively.

d Diffuse.

Table 1.—Contd.

Wave-length	Wave-number <i>in vacuo</i>	Wave-length	Wave-number <i>in vacuo</i>
4415.82 (— 2)	22639.51	4309.25 (o)	23199.39
4413.75 (— 3)	22650.13	4301.98 (— 1)	23238.60
4413.30 (— 3)	22652.44	4301.35 (— 3)	23242.00
4412.07 (— 2)	22658.75	4299.34 (— 3)	23252.86
4411.29 (— 3)	22662.76	4298.41 (— 3)	23257.90
4409.31 (— 3)	22672.95	4296.26 (— 3)	23269.54
4405.85 (— 3)	22690.71	4296.656 (1)	23272.81
4405.16 (— 3)	22694.30	4295.13 (— 3)	23275.66
4404.483 (o)	22697.15	4294.47 (— 3)	23279.23
4401.95 (o)	22710.84	4288.69 (— 1)	23310.61
4397.51 (— 3)	22733.77	4285.85 (— 1)	23326.06
4396.37 (— 3)	22739.67	4283.40 (— 3)	23339.40
4395.17 (— 2)	22745.88	4278.09 (— 1)	23368.35
4390.07 (— 3)	22772.30	4277.95 (— 3)	23369.31
4388.46 (— 3)	22780.65	4277.70 (— 2)	23370.49
4387.53 (— 3)	22785.49	4277.19 (— 2)	23373.28
4384.07 (— 2)	22803.47	4276.79 (— 3)	23375.47
4383.72 (— 3)	22805.29	4274.61 (— 2)	23387.39
4377.77 (— 3)	22836.29	4274.08 (— 2)	23390.28
4377.68 (— 2)	22836.76	4272.08 (— 2)	23401.24
4376.00 (— 3)	22845.52	4270.50 (— 3)	23409.89
4374.81 (— 2)	22851.73	4269.57 (o) D	23415.00
4374.42 (— 1)	22853.77	4269.06 (— 3)	23417.70
4374.29 (— 2) T	22854.45	4268.76 (— 3)	23419.43
4372.46 (— 1)	22864.02	4262.18 (— 3)	23455.59
4369.54 (— 3)	22879.30	4259.65 (— 1)	23469.52
4369.31 (— 3)	22880.50	4259.22 (— 3)	23471.88
4369.03 (— 1) T	22881.97	4258.76 (— 1)	23474.42
4367.16 (— 2)	22891.76	4254.74 (— 3)	23496.60
4366.94 (o)	22892.92	4252.62 (o) d	23508.31
4366.58 (— 2)	22900.04	4252.13 (— 2)	23511.58
4364.05 (— 2) T	22908.08	4251.45 (— 3)	23514.79
4362.87 (o) T	22914.28	4250.48 (— 2)	23520.16
4361.65 (— 1)	22920.68	4250.28 (— 1)	23521.26
4360.89 (— 2)	22924.68	4244.11 (— 3)	23555.45
4360.47 (o)	22926.88	4243.66 (— 3)	23557.95
4356.05 (o)	22950.15	4239.58 (— 3)	23580.62
4352.19 (o) d	22970.50	4238.87 (— 3) T	23584.62
4352.07 (— 1) d	22971.14	4236.33 (— 2)	23598.71
4351.28 (— 2)	22975.30	4235.54 (— 3)	23603.16
4350.838 (1) T	22977.63	4234.01 (— 3)	23611.04
4347.85 (— 2)	22993.43	4225.09 (— 3)	23656.46
4347.52 (o)	22995.18	4221.14 (— 3)	23683.63
4346.36 (— 1)	23001.32	4215.32 (— 3)	23716.23
4345.73 (— 1) T	23004.65	4215.08 (— 3)	23717.68
4341.40 (— 2)	23027.60	4206.84 (— 3)	23764.14
4336.76 (— 2)	23052.23	4201.23 (— 3)	23795.87
4332.44 (— 2)	23075.21	4191.23 (— 2)	23852.65
4328.95 (— 2)	23093.81	4190.19 (— 3)	23858.57
4327.08 (— 2)	23103.79	4185.65 (— 3)	23884.45
4325.77 (— 3)	23110.80	4185.04 (— 2)	23887.92
4325.43 (— 3)	23112.62	4183.55 (— 2)	23896.44
4322.86 (— 3)	23126.35	4183.20 (— 3)	23898.43
4321.00 (— 1)	23136.31	4181.88 (— 3)	23905.98
4319.89 (— 1)	23142.25	4178.38 (— 3)	23926.00
4319.63 (— 2)	23143.63	4176.82 (o)	23934.94*
4311.47 (— 2)	23187.45	4175.93 (— 3)	23940.03

* Possibly a ghost.

Table 1.—Contd.

Wave-length	Wave-number <i>in vacuo</i>	Wave-length	Wave-number <i>in vacuo</i>
4172.34 (— 3)	23960.63	4117.15 (— 1)	24281.81
4171.02 (o)	23968.22†	4113.96 (— 2)	24300.65
4167.98 (— 2)	23985.69	4113.71 (— 1)	24302.13
4166.14 (— 3) D	23996.29	4113.23 (— 2)	24304.96
4164.79 (— 3)	24004.07	4110.58 (— 1)	24320.63
4164.00 (— 3)	24008.62	4105.45 (— 2) <i>d</i>	24351.02
4160.18 (— 3)	24030.67	4104.27 (— 2)	24358.02
4159.58 (— 3)	24034.14	4102.40 (o)	24369.11
4157.55 (— 3)	24045.87	4100.87 (— 2)	24378.21
4153.16 (— 2)	24071.28	4097.98 (o)	24395.41
4148.91 (— 3)	24095.94	4087.46 (— 2)	24458.19
4147.11 (— 2)	24106.40	4086.73 (— 1)	24462.56
4144.71 (— 3)	24120.36	4086.51 (— 1)	24463.88
4140.74 (— 3)	24143.49	4082.91 (— 2)	24485.45
4140.18 (— 3)	24146.76	4077.72 (— 1)	24516.61
4139.46 (— 1)	24150.90	4055.68 (— 3)	24649.84
4136.20 (— 2)	24169.98	4051.61 (— 3)	24674.60
4134.65 (— 3)	24179.05	4047.19 (— 1)	24701.55
4128.70 (— 3)	24213.89	4046.18 (— 3)	24707.72
4125.66 (— 3)	24231.73	4045.99 (— 3)	24708.87
4124.40 (— 3)	24239.14	4044.114 (1)	24720.34
4123.48 (— 3)	24244.55	4042.67 (— 3)	24729.17
4120.46 (— 2)	24262.31	4042.45 (— 3)	24730.51
4119.72 (— 3)	24266.67	4041.97 (— 3)	24733.45
4119.49 (— 3)	24268.02		

† Possibly a ghost.

Table 2. Lines given by Gale, Monk and Lee but not found by the author.

Wave-length	Wave-number <i>in vacuo</i>	Wave-length	Wave-number <i>in vacuo</i>
4712.805 (o)	21212.87	4287.613 (o)	23316.46
4665.770 (oo)	21426.71	4224.105 (o)	23667.01
4644.905 (o)	21522.96	4216.176 (o)	23711.52
4640.192 (o)	21544.82	4211.133 (oo)	23739.91
4638.012 (o)	21554.94	4210.680 (oo)	23742.47
4625.457 (o)	21613.45	4209.690 (o)	23748.06
4624.731 (o)	21616.85	4205.279 (1)	23772.96
4621.806 (o)	21630.52	4200.893 (o)	23798.35
4604.928 (o)	21709.81	4198.455 (oo)	23811.59
4591.925 (o)	21771.28	4189.091 (oo)	23864.83
4588.833 (o)	21785.95	4182.674 (o)	23901.43
4586.120 (o)	21798.83	4179.879 (o)	23917.32
4579.579 (1)	21829.97	4152.164 (oo)	24077.06
4575.981 (o)	21847.14	4101.768 —	24372.88
4553.954 (1)	21953.14	4101.690 —	24373.34
4526.996 (o)	22083.53	4091.068 (oo)	24436.62
4518.128 (o)	22126.88	4086.284 (o)	24465.23
4517.722 (1)	22128.86	4079.432 (o)	24506.33
4508.793 (oo)	22172.68	4076.806 (o)	24522.08
4503.542 (oo)	22198.54	4076.110 (oo)	24526.29
4466.928 (o)	22380.49	4075.010 (oo)	24532.91
4464.352 (1)	22393.40	4072.341 (o)	24549.00
4461.704 (o)	22406.69	4065.182 (oo)	24592.24
4455.023 (o)	22440.29	4064.173 (o)	24598.35
4427.472 (oo)	22579.93	4060.903 (o)	24618.15
4385.704 (oo)	22794.99	4058.093 (o)	24629.74
4354.181 (o)	22960.00	4040.247 (oo)	24743.97
4316.956 (oo)	23157.98		

REVIEWS OF BOOKS

Handbuch der Experimentalphysik, Herausgegeben von W. WIEN und F. HARMS. Unter Mitarbeit von H. Lenz. Band XXII: "Zeemaneffekt," von E. Back. "Ergebnisse und Anwendungen der Spektroskopie, Ramaneffekt," von G. Joos. Pp. x + 436. (Leipzig: Akademische Verlagsgesellschaft m.b.H., 1929.) Unbound 41 gold marks, bound 42.80 gold marks.

This valuable volume on modern optical spectroscopy, while ostensibly concerned with results and applications, has the very desirable feature that enough of the theoretical groundwork is presented and kept constantly in view to enable the physicist to understand the significance of observational data, even if he be not actively engaged in spectroscopic research. It comprises two separate works, each by an author of high repute and unimpeachable authority.

The part devoted to the Zeeman effect occupies the first 189 pages, and contains six chapters dealing with (1) groundwork and empirical rules, (2) experimental methods and apparatus (electromagnets, spectrographs and light-sources), (3) theory of the Zeeman splitting in relation to the term analysis of atomic spectra, (4) the Paschen-Back effect, which Prof. Back modestly calls the magneto-optic transformation effect, (5) by Prof. Joos, the Zeeman effect in band-spectra of diatomic molecules, and (6) the influence of the magnetic field on the absorption spectra of crystals.

The second part of the book, by Prof. Joos, occupies 232 pages, and consists of eight chapters on atomic and molecular spectra, with an appended chapter on the Raman effect. Chapter 1 deals with the analysis of atomic (line) spectra into multiplets and series, and, briefly, with that of band-spectra of diatomic molecules. More detailed accounts of the structure of line spectra appear in Chapters 2 and 3, and of band spectra in Chapter 4. Intensity relations in both line and band spectra are considered in Chapter 5. Chapter 6 deals with spectroscopic determinations of fundamental quantities such as e , m and h , fine structure of spectra lines, and relativity and gravitation effects. Chapter 7, under the title of Applications of Spectroscopy in Chemistry, deals with several unrelated parts of the subject such as quantitative analysis, the isotrope effect, absorption spectra of liquids and solids, and the determination of heats of dissociation of non-polar molecules from their band spectra. Modern astrophysical applications of spectroscopy form the subject of Chapter 8.

With so much that is excellent, it seems ungracious to criticise anything, but the order of treatment outlined above perhaps calls for some comment. The discussion of band spectra of diatomic gaseous molecules, for example, is not connected and self-contained, but is scattered amongst several chapters in the second part, namely Chapter 1 § 4, Chapter 2 §§ 1-4, Chapter 5 § 5, Chapter 7 §§ 2 and 4 (b), and also Chapter 8 of the first part. In the reviewer's opinion a distinct advantage to the reader would have accrued from a blending of these sections into, say, two chapters following those dealing solely with atomic line spectra and preceding those on the band spectra of polyatomic molecules (including liquid and solids) and the Raman effect.

In some instances the notation employed is inferior to that now used by most (including many German) investigators: e.g. the designation of the upper and lower electronic state of an atom or molecule by the subscripts a and e instead of the usual ' and '' has the disadvantage that logically the a should refer to the upper (') state in emission spectra and to the lower (') state in absorption; and such descriptions as *Anfangs-* and *Endzustand*

and their English renderings, initial and final states, therefore often leads to confusion. Nor is the notation consistent throughout one part of the book; for while on pp. 208 and 304 ν_0 and x appear for the nuclear vibration frequency (for $n = 0$ and for infinitesimal amplitude) and anharmonicity factor respectively, yet in the table on pp. 297-303 these quantities are denoted by ν_a and β_a for the upper state and ν_e and β_e for the lower state, in both emission and absorption systems; the more usual notation for these is ω_0' and x' , and ω_0'' and x'' respectively. On the other hand, the appearance of the symbols Li, Be^I, B⁺⁺, etc., instead of Li I, Be II, B III, etc., is welcome.

In the table of molecular data on pp. 297-303 the information given for some molecules is very recent (1929), while for others, e.g., CO and C₂, it does not represent the state of knowledge early in 1928; and in Fig. 17, p. 294, the omission of a ¹D level of Mg makes the general resemblance between CO and Mg electronic levels appear slightly more complete than it really was, even without the inclusion of the CO systems discovered in and since 1927.

To the list of corrections of errors, given on p. 424, the reviewer is able to add the following: the term $n(\nu_0 - nx + \dots)$ in equations (15) and (16) on p. 208 should be $n\nu_0(1 - nx + \dots)$ and the corresponding term in equation (17) should be $(n + \frac{1}{2})\nu_0 - (n + \frac{1}{2})^2\nu_0x$; and the names of Selwyn and Ruedy are misspelt (pp. 231, 233, 234, 236). All these are, however, at most quite minor blemishes in a very admirable addition to the volumes of this *Handbuch* which have already appeared.

W. J.

Wave-Lengths and Atomic Levels in the Spectrum of the Vacuum Iron Arc, by KEVIN BURNS and FRANCIS M. WALTERS Jr. Pp. 159-211. (Publications of the Allegheny Observatory of the University of Pittsburg, 6, no. 11, 1929.)

Whether or not vacuum sources will ever replace the air-iron arc for general use as standards of wave-length in the ordinary photographic region, they are necessarily so used in the further ultra-violet; and for the latter region new standards may be computed with great accuracy by applying the combination principle to the atomic levels obtained from the precise measurement and classification of lines in the former region. In the first part of this publication quartz interferometer measurements are given for over 700 lines in the vacuum-iron arc from λ 8825-2760 Å., the observed wave-lengths (reduced to 15° C. and 760 mm.) and vacuum wave-numbers being recorded in a table which also includes intensity estimates, classifications (temperature, pressure, and energy level), solar data and the best available wave-lengths for the air-iron arc. The second part deals with the construction of the system of energy levels and the computation of data for further lines; one table shows the values and designations of nearly 300 levels, of which 180 are classified, and another contains the computed data for nearly 1500 lines between λ 9653 and 2015 Å. together with their level notations and comparisons with observed wave-lengths both in the vacuum arc and in the solar spectrum. Using the shorter computed wave-lengths as standards, it will be possible to make exact measurements of the Fe⁺ spectrum, and a similarly computed extension of that spectrum would, it is noted, furnish many good standards in the Schumann region. The authors suggest that the record of any general analysis of an atomic spectrum should be accompanied by an intensity diagram, which may be useful in the understanding not only of the spectrum in question but also of the spectra of allied elements; they give a series of intensity diagrams for neutral Fe and neutral Ti, since those for Ti may be helpful in the classification of the unassigned levels of Fe. The reviewer, in common with many spectroscopists, would like to see the designations M I, M II, M III, . . . abandoned in favour of M, M^I, M⁺⁺ . . . for the neutral, singly ionised, double ionised, . . . atom. Even in this beautifully printed publication Fe I and Ti I exactly resemble chemical formulae for iodides. Single ionisation is conveyed to mind

less easily by Fe II than by Fe⁺; and, to take an extreme case, sextuple ionisation can be denoted much better by a contraction such as Cl⁶⁺ than by the notation Cl VII now commonly used.

W. J.

Infra-red Analysis of Molecular Structure, by F. I. G. RAWLINS and A. M. TAYLOR. Pp. xv + 176. (Cambridge Series of Physical Chemistry: Cambridge University Press, 1929.) Price 10s. 6d. net.

The authors remark that no book in English exists which deals with the application of infra-red spectroscopy to the analysis of molecular structure and they offer their work to those undertaking investigations in molecular physics who approach the subject from the point of view of the mathematician or of the physical chemist. They discuss, naturally rather incompletely, both the theory and the experimental side of the subject, in both cases with a great measure of success. The book deals with gases, liquids (including solutions), solids, experimental apparatus, and mathematical theory. In the latter the newer investigations of wave-mechanics find a place. In a few cases some irregularities which might well have been removed by the editor occur. The curves of Fig. 8 do not appear to be described in the text in sufficient detail; on p. 57 "W. Mc. C. Lewis" appears instead of W. C. McC. Lewis," and the style occasionally leaves something to be desired, as for example on p. 71, where we are told that "a weapon will be forged which will not only be powerful in revealing the secrets of atomic and molecular linkages, but which may enable a check to be imposed upon the accuracy of deductions made from X-ray analysis." In several cases, in fact, the text shows some signs of haste and lack of adequate revision. The symbol *R* in the equation on p. 97 is not defined. The use of the German words *Eigenwerte* and *Eigenfunktionen* on p. 152 is not very satisfactory since it is hardly likely that, even at Cambridge, the mathematical conceptions they convey are unknown. The subject has moved so rapidly that some parts of the work need to be supplemented by newer accounts, such as that of Mecke and the Faraday Society's Discussion at Bristol, to which both the present authors contributed interesting papers. The book is a very useful introduction to the subject and above all is intelligible to the average physical chemist.

J. R. P.

Speech and Hearing, by HARVEY FLETCHER, Ph.D. Pp. xv + 331, with 151 figures, and an introduction by H. D. ARNOLD, Ph.D. (London: Macmillan and Co., Ltd.) 21s. net.

During the last decade the fundamental research carried out at the Bell Telephone Laboratories into all phases of the hearing of speech and music has led to the publication of a number of valuable papers. The information obtained is of interest in itself, but it is specially important in connexion with the requirements and limitations of various types of equipment for transmitting, recording and reproducing speech and music. The research has also produced indirect results in that the apparatus designed for the research has had far-reaching applications to distortionless gramophone, talking film and radio reproduction. The compilation by Dr Fletcher, the Acoustical Research Director of the Laboratories, of a book crystallising the experiments performed on speech and hearing and the conclusions reached during the investigations will therefore be widely welcomed. Although naturally, in many places, it is closely related to the original papers referred to, it co-ordinates and interprets them with insight and lucidity. It is especially valuable as an authoritative survey from one of the most successful laboratories dealing with this phase of research. Dr Fletcher's high authority for the opinions he expresses mitigates to some extent the absence at times of references to the full literature of the subject, and Dr Arnold's intro-

duction usefully informs us that in the work discussed by Dr Fletcher every effort has been made to use information found by other experimenters.

The book is divided into four distinct parts, each separately subdivided into chapters 1, 2, etc. This subdivision may arise from the fact that the book takes the general form in which it was presented in the Bell Laboratories' programme of "out-of-hours" courses, but it would be a convenience in future editions if cross references were given definitely to pages, rather than to "Part III Chapter II," etc., for there are no part or chapter numbers at the heads of the pages.

A valuable incidental feature is the adoption of the "bel," a power step of 10^1 , and its subdivision the "decibel" ($10^{0.1}$) as units of intensity ratio. Both names have been adopted by telephone engineers. The "decibel" is equal to the well-known "transmission unit" of telephony, and is also approximately the acoustical intensity ratio ordinarily just detectable by the ear. In consequence of this latter characteristic the ratio has been called a "sensation unit," but since it is purely a numerical ratio, and since our knowledge of sensations is not very extensive, it is better to call it a "decibel," and avoid unnecessary entanglements. With two exceptions (Fig. 71 and p. 199, line 10) Dr Fletcher has consistently adhered to this nomenclature throughout the volume.

The book is well printed and well illustrated. It opens with the mechanism of speaking, with references to Sir Richard Paget's work, and an exhaustive study of the wave form and power of speech sounds and of their frequency of occurrence in the English language. It is presumably due to an American influence in pronunciation that the sounds in the words "part, not, and father" are taken to be similar enough to be designated phonetically by a single sign. Acoustic analyses of typical singing voices and musical instruments are given also, together with estimates of the power output—factors which are of importance in the design of faithful reproducing systems.

In a chapter on the increasingly important subject of noise, spectral analyses of certain noises are supplemented by physiological estimates of their loudness. In these estimates various notes of standard pitch were separately adjusted in volume until they were either just drowned by the noise or judged to be equally loud. The recent work of Barkhausen and his collaborators on similar lines is not referred to. It appears from page 244 that attempts to develop universal methods for calculating loudness from a knowledge of the acoustic spectrum of complex tones have so far been unsuccessful, and it follows that a formula put forward in 1925 by Steinberg has not proved to be applicable outside the range of the type of experiments upon which it was based.

A very interesting account is given of the ear and its behaviour, and of the theory of the action of the ear which Dr Fletcher considers to be most in accord with the varied data now available. Key references are given to other theories which have been the subject of well-known controversy for some time but are not described. Dr Fletcher does not discuss the history of the theory set out, but it is largely his own extension, on the lines of Gray and of Roaf, of the Helmholtz theory.

A chapter on various simple methods of testing acuity of hearing, and their interpretation in numerics, affords highly interesting reading.

The section of the book on the perception of speech and music deals with questions of loudness, pitch and intelligibility, and then discusses the adverse effect of distortion by resonances, by reverberation, or by, say, alteration of the speed of gramophone discs, and of the adverse influences of noise and of deafness.

A. H. D.

The Acoustics of Orchestral Instruments and of the Organ, by E. G. RICHARDSON.
Pp. 158. (London: Edward Arnold and Co.) 10s. 6d. net.

This book is based on a series of lectures by the author on the organ and other wind instruments, additional chapters having been inserted to cover strings and the orchestra in

general. The types of instruments described are the flute, reed instruments, brass, percussion and strings, while a concluding chapter deals with the *ensemble*. The subject is treated in a popular style which will appeal to musicians having only an elementary knowledge of physics. Nevertheless the book will be appreciated also by physicists interested in musical instruments, for the principles involved in their operation are clearly described and a considerable amount of new information, derived from recent research, has been included.

The author states that "the organ being in itself an orchestra does not receive a special chapter." The omission of such a chapter is in some respects unfortunate notwithstanding Dr Richardson's contention. Perhaps a little more might have been said about the organ as a musical instrument and a little less about the church bells which, despite their interesting characteristics, are seldom regarded as orchestral instruments.

Reference is made on page 25 to the end correction at an open end of a pipe, amounting to 0.58 of the radius, but the author has omitted to point out that a much larger end correction, at the blown end, must be applied in determining the pitch of an actual organ pipe.

The final chapter dealing with the orchestra as a whole includes a general survey of the important subject of architectural acoustics, and interesting references are made to certain buildings, e.g. the Pleyel Hall in Paris, having noteworthy acoustic properties. The book is written in an interesting style and is well produced. The illustrations are particularly good and well chosen.

Molecular Magnetism, by Prof. W. PEDDIE, D.Sc., F.R.S.E. Pp. viii + 140, with 38 diagrams. (London: Edward Arnold and Co., 1929.) 10s. 6d.

At a time when the attention of physicists is focussed so much on recent advances, and even text-books may have hectic references to the latest papers added in proof, it is a pleasant change to read a book in which due consideration is given to those parts of a subject in which a classical outlook can still be of service. The hero of Prof. Peddie's book is Weber, whose pioneer work on the theory of dia- and paramagnetism well deserves the prominence given to it. Weber's conception of molecular magnets arising from steady or induced molecular currents forms the basis of the whole treatment. With only slight modifications (such as the substitution of electronic currents for Weber's continuous currents) the consequences of Weber's views are developed as far as possible without the introduction of any of those "unexplained arbitrary hypotheses" which, it is suggested, are characteristic of the quantum theory. From a strictly classical standpoint, the trouble is that the physical world itself is very arbitrary. The value of the quantum theory lies in the fact that it is making that world seem less arbitrary. Even without the aid of the quantum theory, however, Prof. Peddie shows that a great deal can be done.

An account is given of paramagnetism, in which the treatment of Langevin is followed, and the effect of the Weiss molecular field is considered, but the main subject-matter of the book is molecular aggregates, crystals, particularly those of ferromagnetics. A straightforward and admirable treatment is given of the complicated problem of calculating the mutual magnetic effects of a number of molecular magnets arranged on a crystal lattice. It is shown that a cubic crystal will not be magnetically isotropic, and the values of the internal fields arising from the magnetic interaction are worked out. The results are in remarkable agreement with those found experimentally for crystals of iron, nickel and other ferromagnetics. Certain conclusions as to the "magnetic lattice," however, may require modification, as the work of Mahajani shows that changes of sign are introduced in some of the expressions if the magnetic elements are regarded as electron orbits rather than as small but finite magnets.

Stress is laid on the small values of the internal fields (of the order of a hundred gauss) which are found theoretically and apparently confirmed by a number of the experimental facts considered. These are contrasted with the enormous values required by the Weiss molecular field hypothesis. The difficulties seem to arise from the fact that there are two distinct types of internal fields to be considered. One of them—that arising from the mutual purely magnetic energy—is comparatively small, but is adequate to account for such effects as the differences in the properties of an iron or nickel crystal in different directions. To account for the distinctive characteristics of ferromagnetics, however, it seems necessary to postulate some interaction, of a non-magnetic origin, which may be very large. As an alternative, Prof. Peddie makes the unorthodox suggestion that the thermal energy associated with the magnetic elements may be very small—that kT may be small compared with the kinetic theory value—but this is equally arbitrary and leads to difficulties in other directions. There is now, moreover, some indication as to the origin of the mysterious Weiss molecular field in Heisenberg's theory of ferromagnetism. Where Prof. Peddie's conclusions are unorthodox, however, an adequate proof or disproof of them would be highly desirable.

The atomic side of magnetism is now fairly well developed, although many details are still obscure. The next step is to attack the problems of atomic and molecular aggregates in crystals. To those who wish to investigate these problems, Prof. Peddie's book will be of great value, for it is on what may be called the complex side of magnetism, the crystal side, that he has concentrated. Students, to whom the book is mainly addressed, will, no doubt, like others, find some parts difficult. It is by no means a purely introductory book, for although some aspects of the subject are touched on but lightly, others are dealt with very thoroughly. These are just those whose treatment in some other books on magnetism has been least adequate. For the special account of crystalline media, and for the systematic treatment of the subject-matter as a whole, with as few assumptions as possible, the book can be strongly recommended to all who are interested in magnetism.

E. C. S.

Geophysics Memoir 47 of the Meteorological Office. Report on Thames Floods and on the Meteorological Conditions associated with them, by A. T. DODSON and J. S. DINES. Pp. 39. (London: H.M. Stationery Office.) 2s. 6d.

These floods are found to be due to the simultaneous occurrence of tidal high water and a storm in the North Sea which produces a "storm surge" of the water to the west. The amount of water flowing down the Thames is not of primary importance. The probable occurrence of floods such as that of January 6-7, 1928, is about 1 in 60 years and would be reduced to about 1 in 150 years if the training walls were raised 1 ft. It does not appear possible to issue useful warnings of such floods with our present limited knowledge of their causes.

C. H. L.

Handbuch der Experimentalphysik, Wien and Harms, Band 8, Theil 1: *Energie und Wärmeinhalt*, by Prof. A. EUCKEN (Breslau). Pp. xvi + 736. Royal 8vo. (Leipzig: Akademische Verlagsgesellschaft m.b.H.) 65 marks.

This volume covers the calorimetry of solids, liquids and gases, latent heats, heats of solution, dilution and chemical action. It is certainly the most complete account of the experimental methods in use which is available in book form. Sufficient theory is given to make the object of the experimental work apparent and it is set down in a clear

and concise way. Good subject and name indexes and a list of the symbols used, with references to the pages on which they are defined, add materially to the ease with which the volume may be consulted. The print, illustrations and general appearance of the volume are excellent, but the high price must have a serious effect on its circulation.

C. H. L.

Materiewellen und Quantenmechanik, by Prof. A. HAAS (Vienna). Pp. 189. 8vo. (Leipzig: Akademische Verlagsgesellschaft m.b.H., 2nd edition, 1929.) 8 marks.

Although it is little more than a year since the first edition of this introduction to wave-mechanics appeared, the rapid advance of the subject has rendered the addition of four new chapters necessary. The treatment is such as to give a general idea of the lines on which the subject has developed and the results which have been attained rather than a summary of the memoirs of de Broglie, Schrödinger, Heisenberg and Dirac. Non-mathematical readers, for whom the preface states that the book is intended, may find five of the chapters heavy and are advised to omit them.

C. H. L.

Tables Annuelles de Constantes et Données Numériques. (Paris: Gauthier-Villars et Cie.)

(1) Données Numériques d'Électricité, Magnétisme et Electrochimie, pp. xii + 488; (2) Données Numériques de Spectroscopie, pp. viii + 363; (3) Données Numériques de Photographie, pp. x + 36; (4) Données Numériques de Radioactivité, pp. vii + 29; (5) Données Numériques de Radioélectricité, pp. vii + 26; (6) Données Numériques sur les Colloïdes, pp. viii + 58; (7) Données Numériques de Cristallographie, pp. viii + 52; Données Numériques de Physique du Globe, pp. vii + 59; (9) Art de l'Ingénieur et Métallurgie, pp. xiv + 169.

Dr Marie and his international Committee continue with praiseworthy regularity the good work of keeping active researchers in the physical and allied sciences in close touch with recently published numerical data in almost every branch of physics. These volumes form an almost indispensable supplement to the tables of Landolt and Bornstein and to the International Critical Tables, and they have the advantage that they may be obtained in handy sections of some fifty or a hundred pages each, so that workers in a particular subject need not be put to the expense of buying a volume containing data in which they take but little interest.

The volumes constitute parts of Volume 6, published in 1928 but containing data for the years 1923-1924.

The contents of the volumes are briefly as follows: (1) Electric conductivity, insulation, magnetism, atomic moments, electrolytes, etc. (2) This is a large volume which includes emission spectra, the Zeeman effect, X-ray spectra and band spectra. Much difficulty and confusion may arise from ever-changing notations and it is therefore interesting to note that in the record of Fowler's work on carbon, a Bohr notation is employed in the tables with the corresponding Rydberg numbers in parentheses. (3) Is a volume very useful to those physicists who rely on the photographic plate, and contains many data upon blackening of plates, grain distribution and the properties of photographic materials. (4) Includes, in addition to radioactivity, information concerning the ionisation of gases, thermionics, X-ray diffraction and isotopes. (5) Radioelectricity is the elegant name for "wireless," and the volume includes such topics as valves, attenuation coefficients, atmospherics, and finally a table for the calculation of inductances. (6) Includes the properties of gels and the adsorption of gases and dissolved substances. (7) Includes data upon X-ray determinations of crystal structure. (8) Includes radioactivity of the rocks, solar radiation

and meteorology. (9) Includes the properties of building materials, glass, enamels, rubber, lubricants and fuels. Under metallurgy are given the conductivities and mechanical properties of metals and their alloys.

There is no indication of the subscription price.

J. E. C.

Grundriss der Physiologie für Studierende und Ärzte, von CARL OPPENHEIMER und OTTO WEISS. Erster Teil: *Biochemie*, by Prof. CARL OPPENHEIMER. Pp. 420 + viii, with 8 figures in text, name and subject indexes, but no bibliography. (Leipzig: Georg Thieme, 6th edition, 1929.) 17 marks, unbound.

Prof. Oppenheimer has divided this edition of his introduction to the study of biochemistry into two parts. He first deals systematically with the chemical composition of the animal body, and the chemical and physical properties of the substances found therein. In the same part he then considers enzymes, antigens and antibodies—biological units whose chemical structures have not yet been elucidated—in terms of their activities. Of interest to the physicist will be the sections on the colloidal state of proteins, and on enzymes as colloids. In the second part, the chemical functions of tissues and organs are discussed. Accounts are given of changes in the chemical composition and physical states of matter and of the energy transformations associated with physiological activities such as muscular work, secretion, excretion, absorption and the transport of materials.

For those who wish to obtain a brief account of the biochemical and biophysical approach to physiology in the German language, this book can be recommended. It is as up-to-date as can be expected of any general text-book on a subject which grows as rapidly as biochemistry.

M. T.

Lehrbuch der Enzyme: Chemie, Physikalische Chemie, und Biologie, by Prof. CARL OPPENHEIMER. Pp. ix + 660 with 18 figures, subject index, and a few scattered references. (Leipzig: Georg Thieme, 1927.) 33 marks, unbound.

One of the objectives of the biochemist, in his analysis of the chemical processes occurring in living organisms, is the isolation from living tissues of non-living catalytic systems which will show the same activity outside the organism as they did when forming part of the living machinery. These biochemical catalysts are called enzymes. Most of the chemical changes occurring in organisms do so under the influence of specific enzymes. Every month sees an addition to the number of enzymes which have been separated in an active form from living tissues.

Experiments on the separated enzyme systems can take many forms, and these are indicated by the subject matter of the book under review. In Part I, Prof. Oppenheimer records what is known about the chemical nature of enzymes: but very little is known, for direct analysis of extracted systems has not, so far, been successful in isolating pure chemical compounds which show the same activity as the systems. But there is little doubt that most enzymes are colloidal catalysts, and as such, if their chemical nature be disregarded, they may be investigated by the physico-chemical methods used in experiments on colloidal dispersoids, and on catalyses in heterogeneous systems. A useful account of the physical chemistry and kinetics of enzymes has been written by Dr Richard Kuhn in a special chapter of about 90 pages: he first briefly considers electrical charges on, surface activity and adsorption relations and stability of, enzyme systems; then he treats reaction velocities under different conditions for e.g. varying temperatures—enzymes are thermolabile—and varying $[H^+]$ to which most enzymes are sensitive; finally the kinetics of eleven

enzyme entities, when in activity, are shortly reviewed. In Part 2, Prof. Oppenheimer analyses systematically the chemical changes which occur under the influence of enzymes outside and inside living organisms. A very great deal of valuable biochemical information is concentrated in this part.

This work may be described as a good critical summary of our knowledge of enzymes up to 1927.

M. T.

Examples in Elementary Physics, by S. R. HUMBY and F. W. GODDARD. Pp. viii + 166. (London: Longmans, Green and Co., Ltd.) 2s. 6d.

This small volume is arranged to cover the syllabuses of the various school certificate and matriculation examinations, and the greater part of the First M.B. and Army Entrance examinations.

The examples are well chosen, and are representative of all the great branches of physical science. They are some hundreds in number, and for the most part possess the merit that the arithmetic involved may be simplified by cancelling. The book is well produced, is modest in price and can be thoroughly recommended.

A. F.

